1.6. Other types of Equations

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Objective 1: Solve Polynomial Equations by factoring.

Objective 2: Solve Radical Equations

Objective 3: Solve Equations that are quadratic in form

1 Polynomial Equations

E.g. $4x^4 = 12x^2$

Step 1: Get 0 on right side:

$4x^4 - 12x^2 = 0$ (Subtract $12x^2$ from both sides)

Step 2: Factor

$4x^2(x^2 - 3) = 0$ (Factor out the common factor $4x^2$)

Step 3: Set each factor equal to zero and solve:

$4x^2 = 0$ or $x^2 - 3 = 0$

$\rightarrow x^2 = \frac{0}{4}$ $\rightarrow x^2 = 3$

$\rightarrow x^2 = 0$ $\rightarrow \boxed{x = \pm\sqrt{3}}$

$\rightarrow \boxed{x = 0}$

Solution set: $\{0, \sqrt{3}, -\sqrt{3}\}$

E.g. Factor by grouping method:

$$x^3 + x^2 = 4x + 4$$

Step 1: Get 0 on right side:

$$\underbrace{x^3 + x^2}\ \underbrace{-4x - 4} = 0$$

(Subtract $4x + 4$ from both sides)

Step 2: Factor by grouping:

$$\underbrace{x^2\boxed{(x+1)}}_{\text{Factor out } x^2 \text{ from the first 2 terms}}\ \underbrace{-4\boxed{(x+1)}}_{\text{Factor out } -4 \text{ from next 2 terms}} = 0$$

$$\underbrace{(x+1)(x^2-4)}_{\text{Factor out the factor } x+1 \text{ from the 2 groups}} = 0$$

Step 3: Set each factor equal to zero and solve:

$x + 1 = 0$ or $x^2 - 4 = 0$

$\rightarrow \boxed{x = -1}$

$\rightarrow x^2 = 4$

$\rightarrow \boxed{x = \pm\sqrt{4} = \pm 2}$

Solution set : $\boxed{\{-1, 2, -2\}}$

E.g. Factor by grouping method

$$2x^3 + 3x^2 = 8x + 12$$

$$\underbrace{2x^3 + 3x^2}_{\text{Factor out } x^2} \underbrace{- 8x - 12}_{\text{Factor out } -4} = 0$$

(Subtract $8x + 12$ from both sides to get 0 on right side)

$$x^2 \boxed{(2x+3)} - 4 \boxed{(2x+3)} = 0$$

$$(2x+3)(x^2-4) = 0$$

(Factor out $2x+3$ from 2 groups)

$2x + 3 = 0$ or $x^2 - 4 = 0$

$\rightarrow 2x = -3$ $\quad$ $x^2 = 4$

$\rightarrow \boxed{x = -\frac{3}{2}}$ $\quad$ $\boxed{x = \pm\sqrt{4} = \pm 2}$

Solution set : $\boxed{\left\{-\frac{3}{2}, 2, -2\right\}}$

② Radical Equations:

E.g. $\sqrt{2x-1} + 2 = x$

Step 1: Isolate the square root on one side:

$$\sqrt{2x-1} = x - 2 \quad \text{(Subtract 2 from both sides)}$$

Step 2: Square both sides:

$$\left(\sqrt{2x-1}\right)^2 = (x-2)^2$$

Squaring gets rid of square root

$$2x - 1 = (x-2)^2$$

Step 3: Solve the resulting equation.

$$2x - 1 = (x-2)(x-2)$$

$$2x - 1 = x^2 - 2x - 2x + 4 \quad \text{(Distribute)}$$

$$2x - 1 = x^2 - 4x + 4 \quad \text{(Combine like terms)}$$

$$-2x + 1$$

$$0 = x^2 - 6x + 5 \quad \text{(Subtract } 2x-1 \text{ from both sides)}$$

$$0 = (x-1)(x-5) \quad \text{(Factor)}$$

$x - 1 = 0$ or $x - 5 = 0$

$x = 1$; $x = 5$

Step 4: Check solutions:

Original equation: $\sqrt{2x - 1} + 2 = x$

Check: $\boxed{x = 1}$ $\sqrt{2(1) - 1} + 2 \stackrel{?}{=} 1$

$\sqrt{1} + 2 \stackrel{?}{=} 1$

$1 + 2 = 1$ False

Extraneous solution

So, $x = 1$ is NOT a solution to the original equation.

Note: An extraneous solution is a solution to the resulting equation (after squaring) but it is NOT a solution to the original equation.

Check: $\boxed{x = 5}$: $\sqrt{2(5) - 1} + 2 = 5$

So, $x = 5$ is a solution to the original equation

$\sqrt{9} + 2 = 5$

$3 + 2 = 5$ True

Conclusion: Solution set to the original equation:

$$\{5\}$$

E.g. Solve: $\sqrt{x+3} + 3 = x$

$$\sqrt{x+3} = x - 3 \quad \text{(Isolate the square root)}$$

$$\left(\sqrt{x+3}\right)^2 = (x-3)^2 \quad \text{(Square both sides)}$$

$$x + 3 = x^2 - 6x + 9$$

$$-x \quad -3$$

$$0 = x^2 - 7x + 6$$

$$0 = (x-1)(x-6)$$

$$x - 1 = 0 \quad \text{or} \quad x - 6 = 0$$

$$x = 1 \quad ; \quad x = 6$$

Check our solutions:

$\boxed{x = 1}$: $\underbrace{\sqrt{1+3}}_{2} + 3 \stackrel{?}{=} 1$

Extraneous

$2 + 3 = 1$ False

$\boxed{x = 6}$: $\underbrace{\sqrt{6+3}}_{3} + 3 = 6$

Solution

$3 + 3 = 6$ True

Solution set: $\boxed{\{6\}}$

3 Equations that are quadratic in form

E.g. Solve: $\overset{u^2}{\boxed{x^4}} - 5\overset{u}{\boxed{x^2}} + 6 = 0$

Method of u-substitution:

Let $u = x^2$. Then: $u^2 = (x^2)^2 = x^4$

Rewrite in terms of u:

$$\boxed{u^2 - 5u + 6 = 0}$$ quadratic form

$(u-2)(u-3) = 0$ (Factor)

$u - 2 = 0$ or $u - 3 = 0$

$u = 2$; $u = 3$

Solve for x:

$x^2 = 2$; $x^2 = 3$

$x = \pm\sqrt{2}$; $x = \pm\sqrt{3}$

Solution set: $\{\sqrt{2}, -\sqrt{2}, \sqrt{3}, -\sqrt{3}\}$

E.g. $(x^2-4)^2 + (x^2-4) - 6 = 0$

Substitution: Let $u = x^2 - 4$

Rewrite the equation in terms of u:

$u^2 + u - 6 = 0$ → quadratic equation

$(u+3)(u-2) = 0$

$u + 3 = 0$ or $u - 2 = 0$

$u = -3$; $u = 2$

Solve for x:

$x^2 - 4 = -3$ $x^2 - 4 = 2$

$x^2 = 1$ $x^2 = 6$

$x = \pm\sqrt{1} = \pm 1$; $x = \pm\sqrt{6}$

Solution set: $\boxed{\{1, -1, \sqrt{6}, -\sqrt{6}\}}$