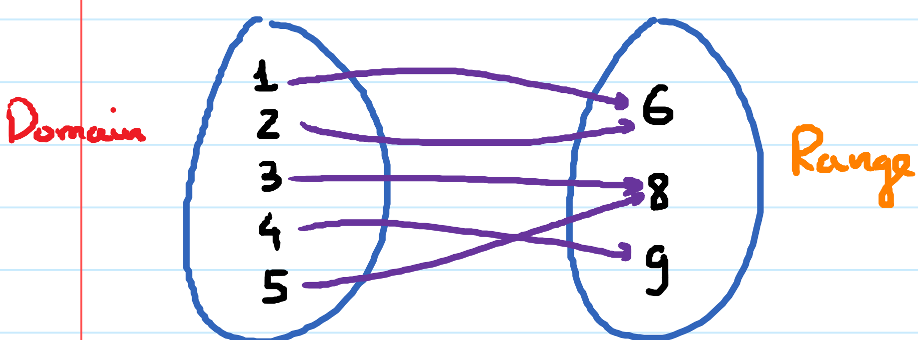
Definition of a function:

Def 1: A function is a relation in which no two ordered pairs have the same first component and different second components (first components cannot be repeated)

Def 2: Equivalently, a function is a correspondence from a first set called the Domain to a second set called the Range such that each element in the domain corresponds to exactly one element in the range.

E.g. Determine whether a relation is a function.

ⓐ $\{(1,6),(2,6),(3,8),(4,9),(5,8)\}$



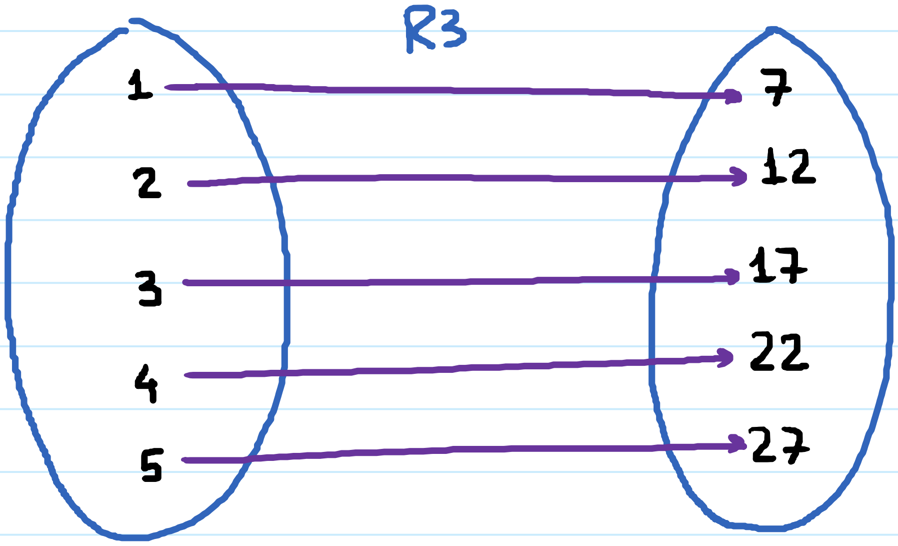
This is a function

(b) $\{(10, 5); (9, 4.5), (8, 4), (7, 3.5), (6, 3), (6, 2)\}$ (6 is repeated)

This is NOT a function (Repetition in first components)

Obj 3: Functions as Equations

R3



This is a function

Domain

Range

x represents the # of tickets

y represents the cost of buying x tickets.

The equation $y = 5x + 2$ gives us the procedure to calculate the cost of buying x tickets.

This equation defines $\boxed{y}$ as a function of $\boxed{x}$.

dependent variable

independent variable

Note: Not all equations in y and x define y as a function of x.

E.g.

$$x^2 + y^2 = 4$$

$$y^2 = 4 - x^2$$

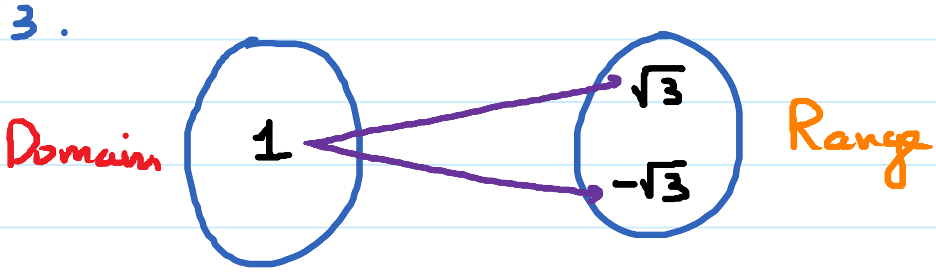
$$y = \pm\sqrt{4 - x^2}$$

This equation does not define y as a function of x.

Reason: There are values of x that correspond to 2 values of y.

For example, if $x = 1$; then $y = \pm\sqrt{4 - 1^2}$

So, $y = \pm\sqrt{3}$.



Note: If an equation is solved for y and more than one values of y can be obtained from a value of x, then that equation does not define y as a function of x.

E.g. Solve each equation for y and determine whether the equation defines y as a function of x.

(a) $2x + 3y = 6$ (b) $2x^2 + 3y^2 = 1$

Solution:

(a) $2x + 3y = 6 \rightarrow 3y = 6 - 2x$

$\rightarrow y = \dfrac{6-2x}{3}$. This defines y as a function of x.

(b) $2x^2 + 3y^2 = 1 \rightarrow 3y^2 = 1 - 2x^2$

$\rightarrow y^2 = \dfrac{1-2x^2}{3} \rightarrow y = \pm\sqrt{\dfrac{1-2x^2}{3}}$

This does not define y as a function of x.