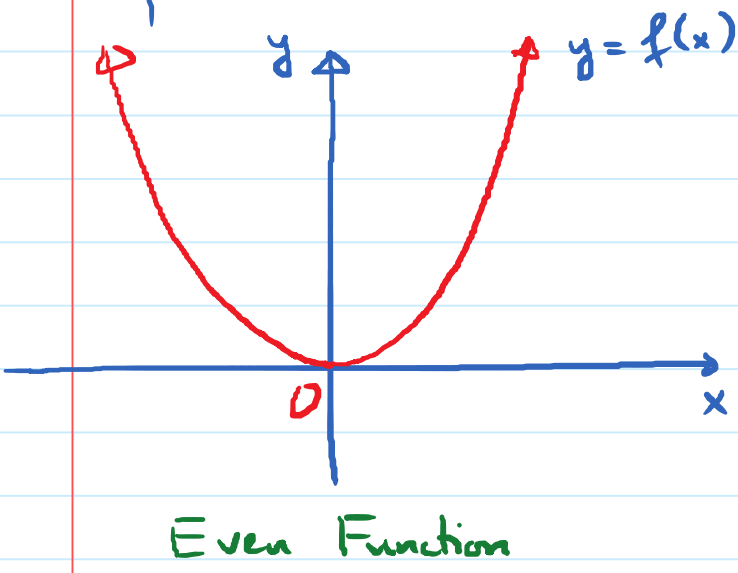
2.2. More on Functions and their graphs.

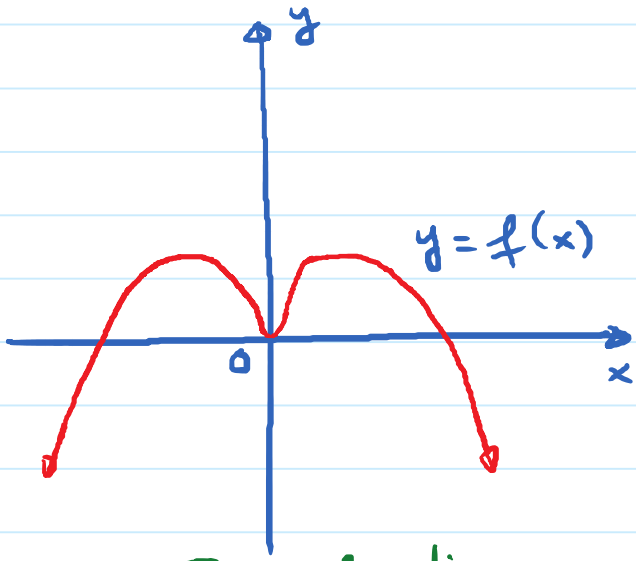
Thursday, January 30, 2020 10:09 AM

Objective 1: Even functions and Odd Functions

Definition: A function whose graph is symmetric with respect to the y-axis is called an EVEN function.



Even Function

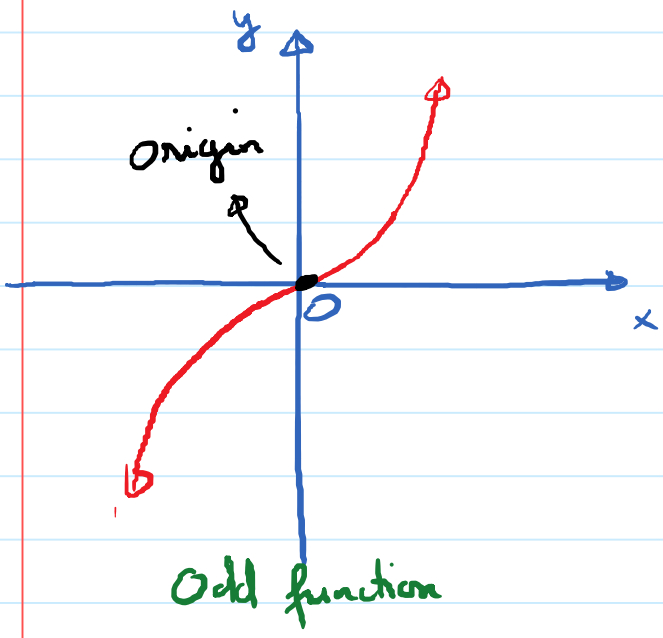


Even function.

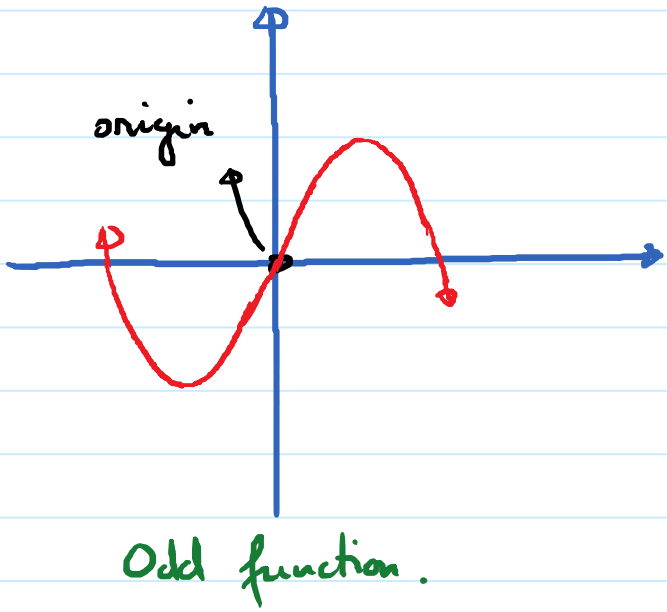
Note: If $y = f(x)$ is an even function, then:

$$f(-x) = f(x)$$

Definition: A function whose graph is symmetric with respect to the origin is called an ODD function



Odd function



Odd function.

Note: If a function is odd, then:

$$f(-x) = -f(x)$$

* How to determine whether a function $y = f(x)$ is odd or even or neither given the equation for $f(x)$?

Step 1: Find $f(-x)$ by replacing x by $-x$ in the equation for $f(x)$.

Step 2: Simplify and compare the equation for $f(x)$ and the equation for $f(-x)$

* If $f(-x) = f(x)$, then f is Even.
* If $f(-x) = -f(x)$, then f is Odd.
* If neither of the above holds, then the function f is neither odd nor even.

E.g. Determine whether the given function is even or odd or neither.

(a) $f(x) = x^4 - 2x^2$.

$f(-x)$ and $f(x)$ are the same

Step 1: Find $f(-x)$: $f(-x) = (-x)^4 - 2(-x)^2$

Step 2: Simplify $f(-x)$: $f(-x) = x^4 - 2x^2$

Compare $f(-x)$ and $f(x)$, we see that:

$$f(-x) = f(x)$$

Conclusion: f is an Even function.

$f(-x)$ is the opposite of $f(x)$

(b) $f(x) = x^3 - 6x$

Step 1: Find $f(-x)$: $f(-x) = (-x)^3 - 6(-x)$

Step 2: Simplify $f(-x)$: $f(-x) = -x^3 + 6x$

Compare $f(-x)$ and $f(x)$, we see that

$$f(-x) = -f(x).$$

So, f is an Odd function

neither same nor opposite

(c) $f(x) = x^2 + 2x - 1$

Step 1: find $f(-x)$: $f(-x) = (-x)^2 + 2(-x) - 1$

Step 2: Simplify: $f(-x) = x^2 - 2x - 1$

Conclusion: f is neither odd nor even.

Objective 2: Piecewise Functions:

E.g.

$$f(x) = \begin{cases} 3x + 5 & \text{if } x < 0 \\ 4x + 7 & \text{if } x \geq 0 \end{cases}$$

(a) $f(-2)$ — -2 is less than 0 → use first formula

$f(-2) = 3(-2) + 5 = \boxed{-1}$.

(b) $f(3)$ — 3 is greater than 0 → use second formula

$f(3) = 4(3) + 7 = \boxed{19}$

(c) $f(0)$ — equal to 0 → use second formula

$f(0) = 4(0) + 7 = \boxed{7}$

E.g. Graph the function:

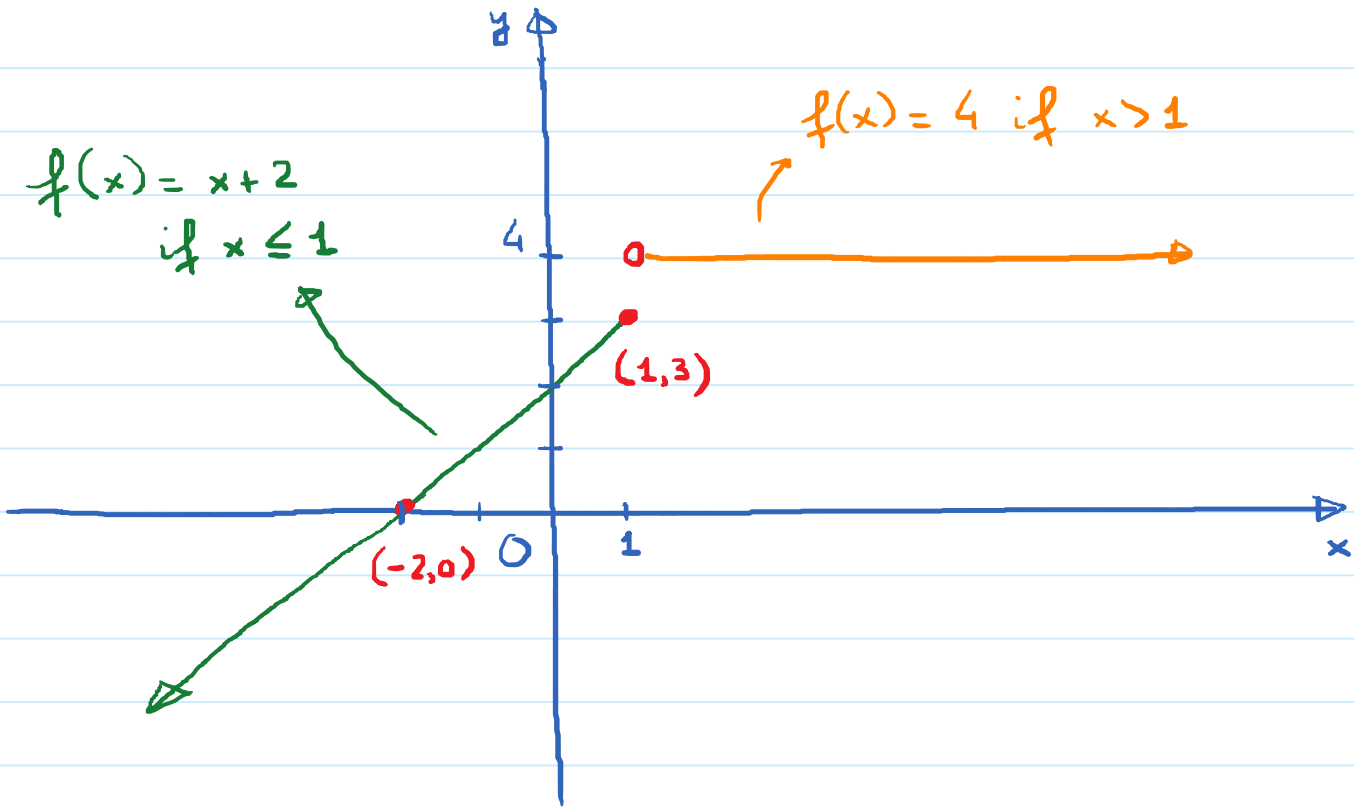
$$f(x) = \begin{cases} x+2 & \text{if } x \leq 1 \\ 4 & \text{if } x > 1 \end{cases}$$

Graph each formula:

1st formula: $f(x) = x+2$ $(\text{if } x \leq 1)$

x	$y = f(x) = x+2$	ordered pair
-2	$y = -2+2 = 0$	$(-2, 0)$
1	$y = 1+2 = 3$	$(1, 3)$

2nd formula: $f(x) = 4$ $(\text{if } x > 1)$



E.g. Evaluate the function at the given value:

(a) $$g(x) = \begin{cases} x+3 & \text{if } x \geq -3 \\ -(x+3) & \text{if } x < -3 \end{cases}$$

Find: $g(0) = 3$ $g(-6) = 3$ $g(-3) = 0$

(b) $$h(x) = \begin{cases} \dfrac{x^2-9}{x-3} & \text{if } x \neq 3 \\ 6 & \text{if } x = 3 \end{cases}$$

Find $h(5) = 8$; $h(0) = 3$; $h(3) = 6$