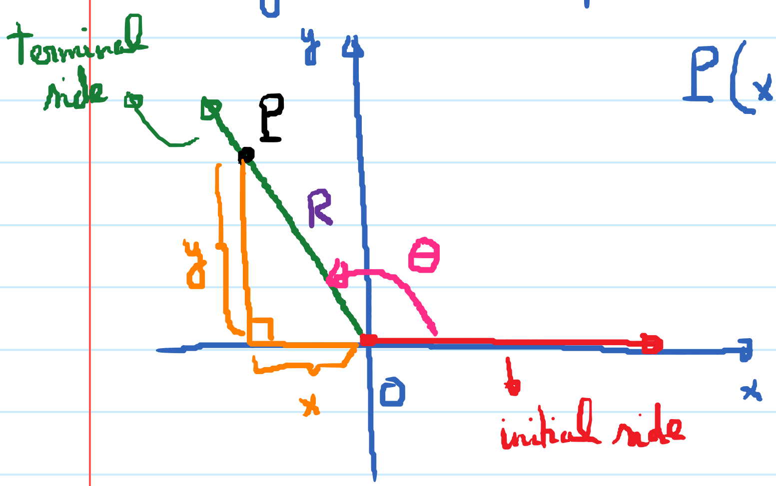
1.3. Trigonometric Functions

Wednesday, January 22, 2020 9:45 AM

θ: angle in standard position.



$P(x,y)$ is any point on terminal side

R: distance from O to P

Pythagorean Theorem:

$$x^2 + y^2 = R^2$$

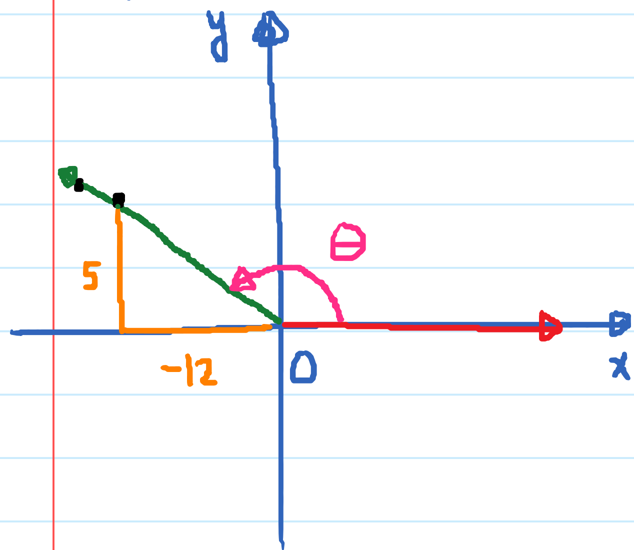
$$R = \sqrt{x^2 + y^2} \quad ; \quad R > 0$$

Definition of the 6 basic trigonometric functions:

$\sin\theta$ (sine of theta) $= \dfrac{y}{R}$; $\cos\theta$ (cosine of theta) $= \dfrac{x}{R}$; $\tan\theta$ (tangent of theta) $= \dfrac{y}{x}$ $(x \neq 0)$

$\csc\theta$ (cosecant) $= \dfrac{R}{y}$ $(y \neq 0)$; $\sec\theta$ (secant) $= \dfrac{R}{x}$ $(x \neq 0)$; $\cot\theta$ (cotangent) $= \dfrac{x}{y}$ $(y \neq 0)$

E.g.1. The terminal side of an angle θ in standard position passes through (-12, 5). Find sin θ, cos θ, ...



$$R = \sqrt{x^2 + y^2} = \sqrt{(-12)^2 + (5)^2}$$

$$\boxed{R = 13}$$

$$\sin\theta = \frac{y}{R} = \frac{5}{13}$$

$$\cos\theta = \frac{x}{R} = \frac{-12}{13} = -\frac{12}{13}$$

$$\tan\theta = \frac{y}{x} = \frac{5}{-12} = -\frac{5}{12}$$

$$\csc\theta = \frac{R}{y} = \frac{13}{5}$$

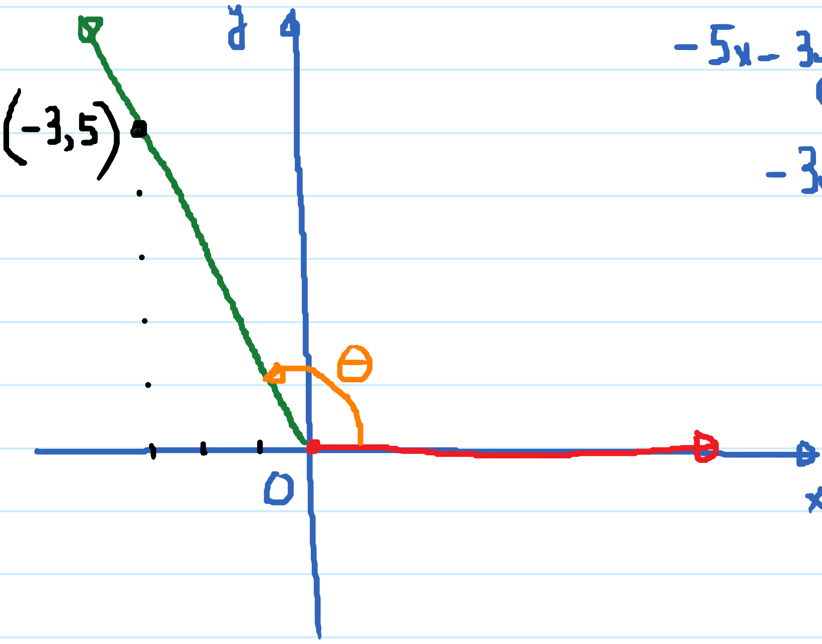
$$\sec\theta = \frac{R}{x} = -\frac{13}{12}$$

$$\cot\theta = \frac{x}{y} = -\frac{12}{5}$$

Note: We can pick any point on the terminal side of the angle and obtain the same result for the values of the 6 trigonometric function.

E.g. Given: Equation of the terminal side of an angle θ in standard position is $-5x - 3y = 0$; $x \leq 0$.

Sketch the terminal side of θ. Find $\sin\theta$, $\cos\theta$, ...



$$-5x - 3y = 0$$

$$-3y = 5x$$

$$y = -\frac{5}{3}x$$

y-int: (0,0)

Slope $= -\frac{5}{3}$ $\left(\frac{\text{Rise}}{\text{Run}}\right)$

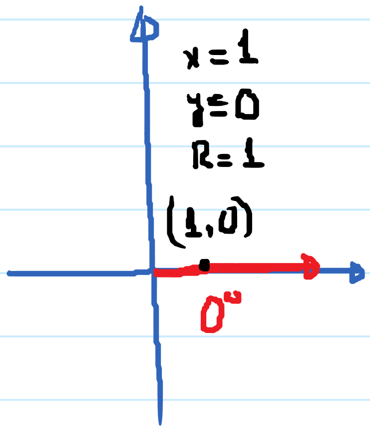
$$R = \sqrt{x^2 + y^2} = \sqrt{9 + 25} = \sqrt{34}$$

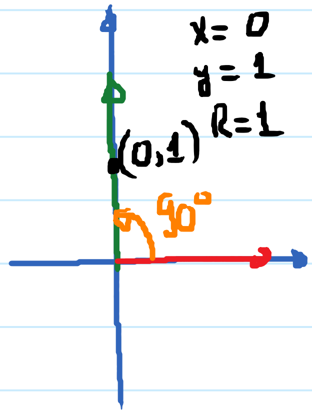
$$\sin\theta = \frac{y}{R} = \frac{5}{\sqrt{34}} \cdot \frac{\sqrt{34}}{\sqrt{34}} = \frac{5\sqrt{34}}{34} \; ; \quad \csc\theta = \frac{\sqrt{34}}{5}$$

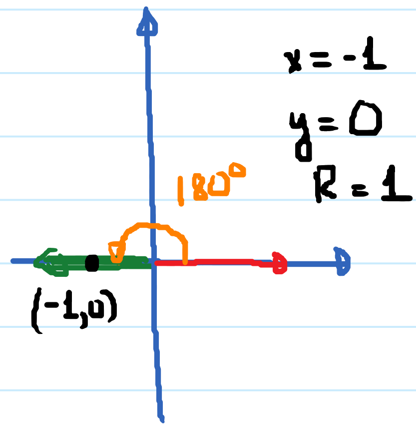
$$\cos\theta = \frac{x}{R} = -\frac{3}{\sqrt{34}} = -\frac{3\sqrt{34}}{34} \; ; \quad \sec\theta = -\frac{\sqrt{34}}{3}$$

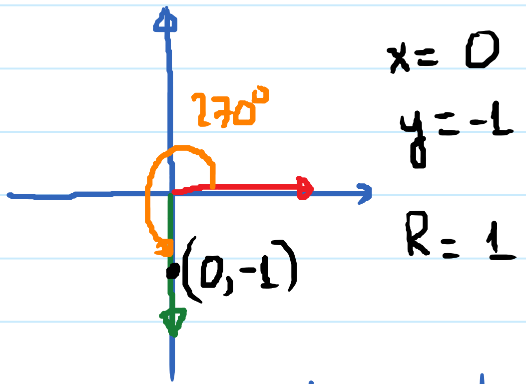
$$\tan\theta = \frac{y}{x} = -\frac{5}{3} \; ; \quad \cot\theta = -\frac{3}{5}$$

Trigonometric function values of the quadrantal angles









θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
0°	0	1	0	undefined	1	undefined
90°	1	0	undefined	1	undefined	0
180°	0	-1	0	undefined	-1	undefined
270°	-1	0	undefined	-1	undefined	0

$\csc 90° = 1$; $\cot 180°$: undefined

Note: Coterminal angles have the same trigonometric function values.

E.g. (a) $\cot 540^\circ \stackrel{?}{=} \cot 180^\circ = \text{undefined}$

$(540^\circ = 180^\circ + 360^\circ$ so 540° is coterminal with $180^\circ)$

(b) $\tan 1800^\circ \stackrel{?}{=} \tan 0^\circ = 0$

$(1800^\circ = 0^\circ + 5\cdot 360^\circ$. So 1800° is coterminal with $0^\circ)$

E.g. $\underbrace{\tan 0^\circ}_{0} - 6\cdot\underbrace{\sin 90^\circ}_{1} = 0 - 6\cdot 1 = -6$

Note: $\sin^2\theta = (\sin\theta)^2$; $\cos^3\theta = (\cos\theta)^3$

$\sin^{2020}\theta = (\sin\theta)^{2020}$

E.g. $\cos^{2020} 180^\circ = (\cos 180^\circ)^{2020} = (-1)^{2020} = \boxed{1}$

$\cos^{2021} 180^\circ = (\cos 180^\circ)^{2021} = (-1)^{2021} = \boxed{-1}$

E.g. $\cos^2(-180^\circ) + \sin^2(-180^\circ) = 1 + 0 = \boxed{1}$.

$(\cos(-180^\circ))^2$ $\quad$ $(\sin(-180^\circ))^2$

$(-1)^2$ $\quad$ $(0)^2$

1 $\quad$ 0