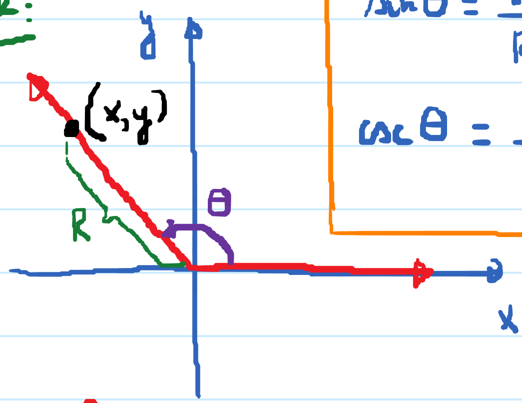
1.4. Using the definition of the trigonometric functions.

Monday, January 27, 2020 9:33 AM

Recall:



$$\sin\theta = \frac{y}{R}; \quad \cos\theta = \frac{x}{R} \quad \tan\theta = \frac{y}{x}$$

$$\csc\theta = \frac{R}{y} \quad \sec\theta = \frac{R}{x} \quad \cot\theta = \frac{x}{y}$$

Reciprocal Identities:

$$\sin\theta = \frac{1}{\csc\theta}; \quad \cos\theta = \frac{1}{\sec\theta}; \quad \tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta}; \quad \sec\theta = \frac{1}{\cos\theta}; \quad \cot\theta = \frac{1}{\tan\theta}$$

E.g. (a) Given that $\sec\theta = 9.804$. Find $\cos\theta$.

$$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{9.804} = 0.101\ldots$$

(b) Given $\sin\theta = -\frac{2}{\sqrt{20}}$, find $\csc\theta$.

$$\csc\theta = -\frac{\sqrt{20}}{2} = -\frac{\sqrt{4\cdot 5}}{2} = -\frac{\sqrt{4}\cdot\sqrt{5}}{2} = -\frac{\cancel{2}\sqrt{5}}{\cancel{2}}$$

So, $\csc\theta = -\sqrt{5}$.

Note: Reciprocals always have the same sign.

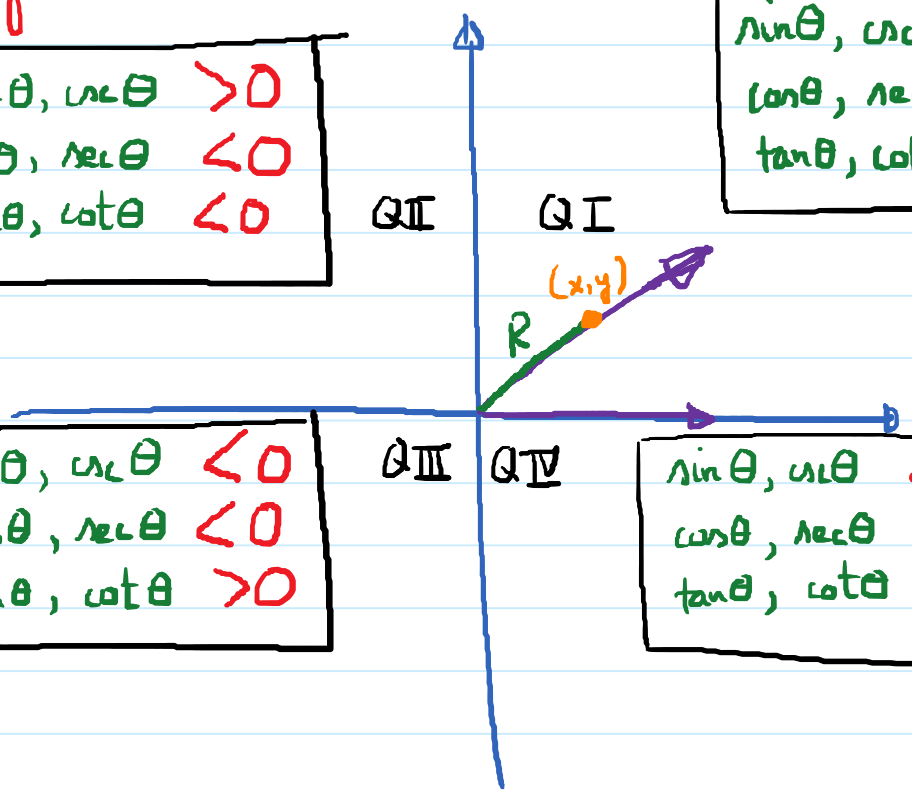
Determine the signs of the trigonometric functions of nonquadrantal angles

QII:

$\sin\theta, \csc\theta > 0$
$\cos\theta, \sec\theta < 0$
$\tan\theta, \cot\theta < 0$

QI:

$\sin\theta, \csc\theta > 0$
$\cos\theta, \sec\theta > 0$
$\tan\theta, \cot\theta > 0$



QIII:

$\sin\theta, \csc\theta < 0$
$\cos\theta, \sec\theta < 0$
$\tan\theta, \cot\theta > 0$

QIV:

$\sin\theta, \csc\theta < 0$
$\cos\theta, \sec\theta > 0$
$\tan\theta, \cot\theta < 0$

E.g. $\cot\theta < 0$ and $\sec\theta < 0$.

Which quadrant does θ belong to? QII

E.g. $\theta = 855°$. Determine the signs of the trig functions of θ.

Coterminal angle: $855° - 2\cdot 360° = 135° \to$ QII

$\sin\theta, \csc\theta > 0$; $\cos\theta, \sec\theta < 0$, $\tan\theta, \cot\theta < 0$

Pythagorean Identities:

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 - \sin^2\theta = \cos^2\theta$$

$$1 - \cos^2\theta = \sin^2\theta$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\sec^2\theta - \tan^2\theta = 1$$

$$\sec^2\theta - 1 = \tan^2\theta$$

$$\cot^2\theta + 1 = \csc^2\theta$$

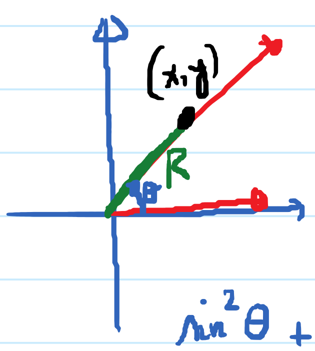
$$\csc^2\theta - \cot^2\theta = 1$$

$$\csc^2\theta - 1 = \cot^2\theta$$

Why?

$$\sin^2\theta + \cos^2\theta = 1 \quad (\text{Why ?})$$

$$\sin\theta = \frac{y}{R} \ ; \quad \cos\theta = \frac{x}{R} .$$



$$\sin^2\theta = \left(\frac{y}{R}\right)^2 = \frac{y^2}{R^2} \ ; \quad \cos^2\theta = \left(\frac{x}{R}\right)^2 = \frac{x^2}{R^2}$$

$$\sin^2\theta + \cos^2\theta = \frac{y^2}{R^2} + \frac{x^2}{R^2} = \frac{y^2 + x^2}{R^2} = 1$$

E.C. Explain why the other 2 identities are true? (+1)

Quotient Identities:

$$\boxed{\tan\theta = \frac{\sin\theta}{\cos\theta} \quad ; \quad \cot\theta = \frac{\cos\theta}{\sin\theta}}$$

Why? $\tan\theta = \dfrac{y}{x}$

$$\frac{\sin\theta}{\cos\theta} = \frac{\frac{y}{R}}{\frac{x}{R}} = \frac{y}{R}\cdot\frac{R}{x} = \frac{\cancel{R}y}{\cancel{R}x} = \frac{y}{x}$$

E.g. Given: $\cos\theta = \dfrac{4}{5}$ and θ is in Q IV

Find $\sin\theta$

Pythagorean identities:

$$\sin^2\theta + \cos^2\theta = 1$$

Since θ is in QIV, the answer must be $\boxed{\sin\theta = -\frac{3}{5}}$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\sin^2\theta = 1 - \left(\frac{4}{5}\right)^2 = \frac{1\cdot 25}{1\cdot 25} - \frac{16}{25} = \frac{9}{25}$$

$$\sin\theta = \pm\sqrt{\frac{9}{25}} = \pm\frac{3}{5}$$

E.g. Given : $\sin\theta = \frac{1}{2}$; θ is in QII.

Find $\tan\theta$.

Pythagorean: $\sin^2\theta + \cos^2\theta = 1$

$$\cos^2\theta = 1 - \sin^2\theta = 1 - \left(\frac{1}{2}\right)^2$$

$$\cos^2\theta = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\cos\theta = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2}$$

θ is in QII, So, $\cos\theta = -\frac{\sqrt{3}}{2}$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = -\frac{1}{2}\cdot\frac{2}{\sqrt{3}} = -\frac{\cancel{2}}{\cancel{2}\sqrt{3}}$$

Quotient identities

$$= -\frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \boxed{-\frac{\sqrt{3}}{3}}$$

Range Values of trigonometric functions (output values)

Function of θ	Range
$\sin\theta$, $\cos\theta$	$[-1, 1]$
$\sec\theta$, $\csc\theta$	$(-\infty, -1] \cup [1, \infty)$

Function	Range
$\tan\theta$, $\cot\theta$	$(-\infty, \infty)$