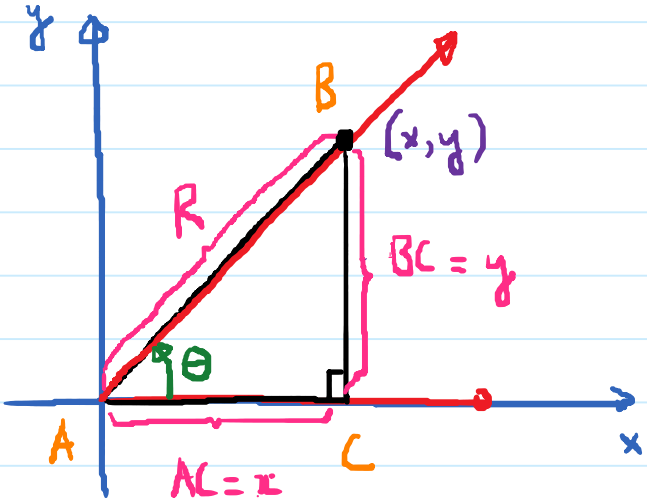
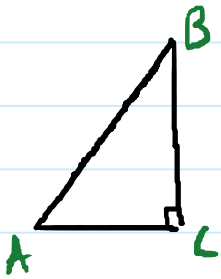
2.1 Trigonometric Functions of Acute Angles.

Wednesday, January 29, 2020 9:30 AM



$\angle A = \theta$



SOHCAHTOA

$\sin A = \frac{opp}{hyp} = \frac{BC}{AB}$

$\cos A = \frac{adj}{hyp} = \frac{AC}{AB}$

$\tan A = \frac{opp}{adj} = \frac{BC}{AC}$

Right triangle - based definition:

$$\sin A = \frac{y}{R} = \frac{BC}{AB} = \frac{\text{side opposite } A}{\text{hypotenuse}}$$

$$\cos A = \frac{x}{R} = \frac{AC}{AB} = \frac{\text{side adjacent to } A}{\text{hypotenuse}}$$

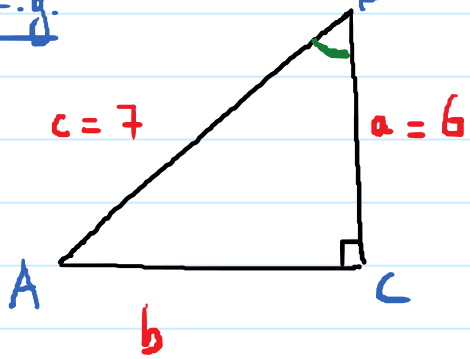
$$\tan A = \frac{y}{x} = \frac{BC}{AC} = \frac{\text{side opposite } A}{\text{side adjacent to } A}$$

$$\csc A = \frac{\text{hypotenuse}}{\text{side opposite } A}$$

$$\sec A = \frac{\text{hypotenuse}}{\text{side adjacent to } A}$$

$$\cot A = \frac{\text{side adjacent to } A}{\text{side opposite } A}$$

E.g.



Find $\sin B = \frac{\sqrt{13}}{7}$ $\cos B = \frac{6}{7}$

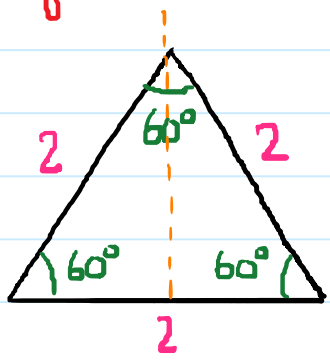
$\csc B = \frac{7}{\sqrt{13}} = \frac{7\sqrt{13}}{13}$ $\sec B = \frac{7}{6}$

$\tan B = \frac{\sqrt{13}}{6}$ $\cot B = \frac{6}{\sqrt{13}} = \frac{6\sqrt{13}}{13}$

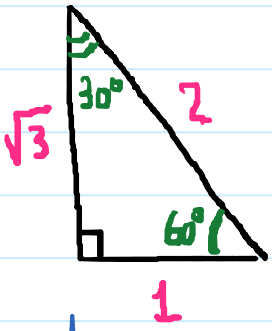
$a^2 + b^2 = c^2 \longrightarrow 36 + b^2 = 49 \longrightarrow b^2 = 49 - 36 = 13$

$b = \sqrt{13}$

Trigonometric Function Values of Special Angles



Equilateral Triangle



$\sin 30° = \frac{1}{2}$; $\csc 30° = 2$

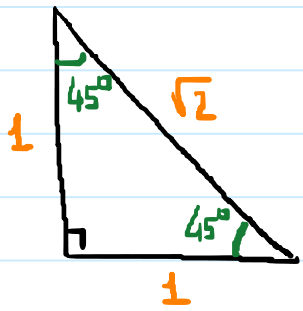
$\cos 30° = \frac{\sqrt{3}}{2}$; $\sec 30° = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\tan 30° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$; $\cot 30° = \sqrt{3}$

$\sin 60° = \frac{\sqrt{3}}{2}$; $\csc 60° = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\cos 60° = \frac{1}{2}$; $\sec 60° = 2$

$\tan 60° = \sqrt{3}$; $\cot 60° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$



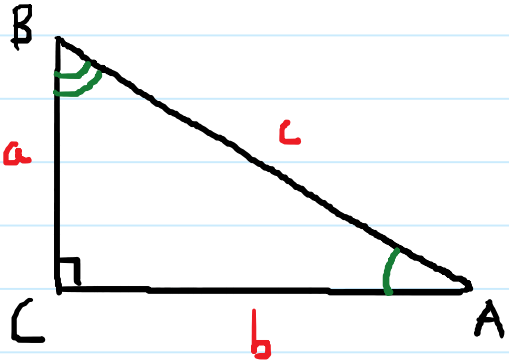
Isosceles Right Triangle

$\sin 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\quad \csc 45^\circ = \sqrt{2}$

$\cos 45^\circ = \frac{\sqrt{2}}{2}$ $\quad \sec 45^\circ = \sqrt{2}$

$\tan 45^\circ = 1$ $\quad \cot 45^\circ = 1.$

Cofunction Identities



$\angle A + \angle B = 90^\circ$

$B = 90^\circ - A$

$\sin A = \frac{a}{c}$; $\cos B = \frac{a}{c}$ $\rightarrow$ $\sin A = \cos B$

$\sin A = \cos(90^\circ - A)$

$\tan A = \frac{a}{b}$; $\cot B = \frac{a}{b}$ $\rightarrow$ $\tan A = \cot B$

$\tan A = \cot(90^\circ - A)$

For any acute angle A:

$\sin A = \cos(90° - A)$; $\cos A = \sin(90° - A)$;

$\tan A = \cot(90° - A)$; $\cot A = \tan(90° - A)$

$\sec A = \csc(90° - A)$; $\csc A = \sec(90° - A)$

E.g. $\sin 9° = \cos 81°$ $\csc 65° = \sec 25°$.

$\cot 76° = \tan 14°$