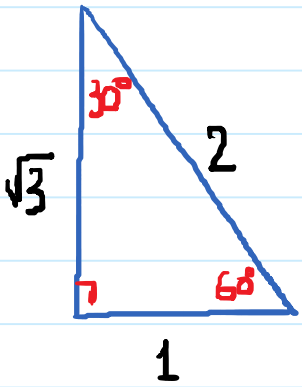
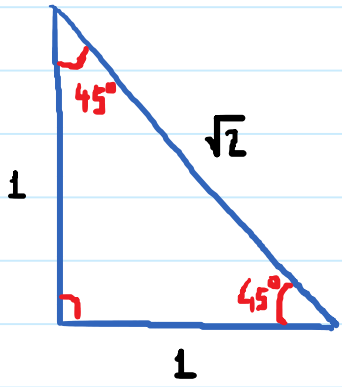
2.2. Trigonometric Functions of Non-Acute Angles

Monday, February 3, 2020 9:37 AM

For acute angles: SOHCAH TOA

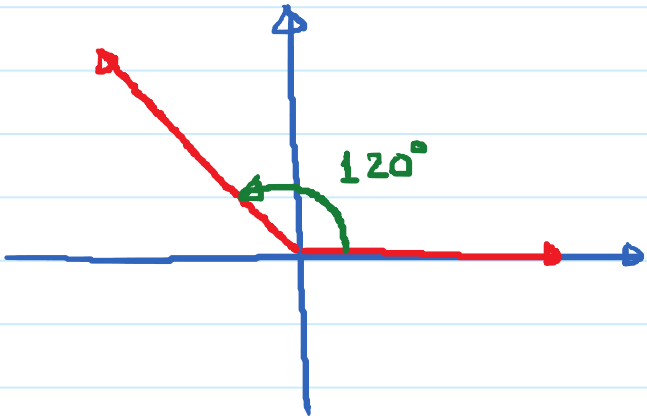
Special triangles:



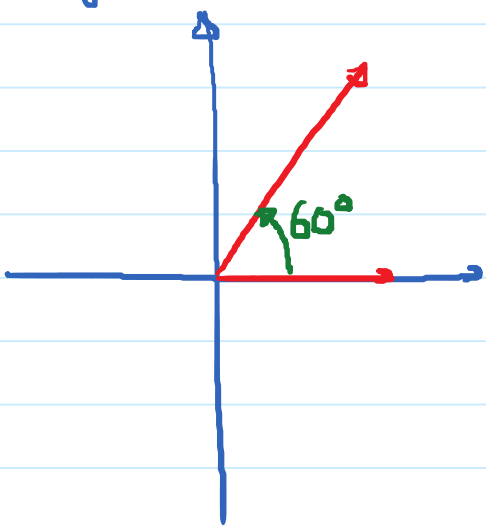


Reference Angle

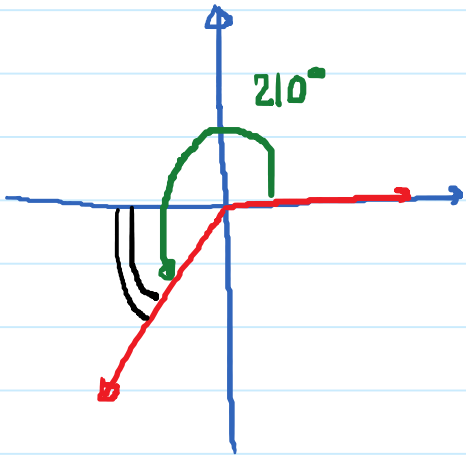
A reference angle is the positive acute angle made by the terminal side of the given angle and the x-axis.



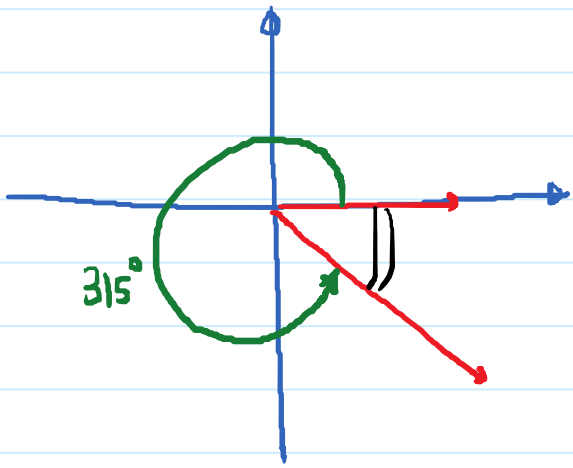
Reference angle = 60°



Reference angle = 60°.



Reference angle = 30°

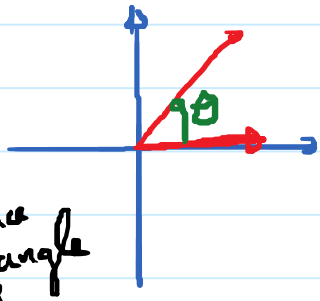


Reference angle = 45°.

To find the reference angle of a given angle

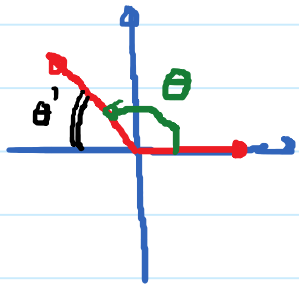
Step 1: If the given angle is NOT already in $[0°, 360°]$, use coterminal angle formula to obtain a coterminal angle θ in $[0°, 360°]$

Step 2: Draw θ to identify the quadrant that the terminal side is in.

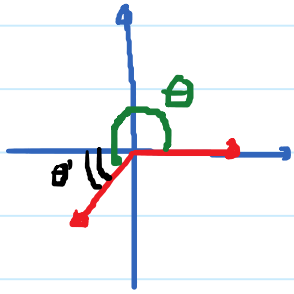


reference angle

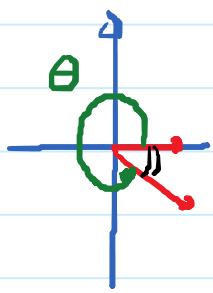
$\theta' = \theta$



$\theta' = 180° - \theta$



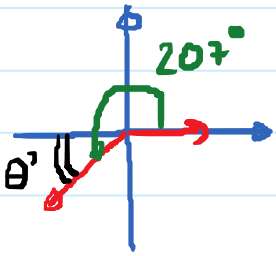
$\theta' = \theta - 180°$



$\theta' = 360° - \theta$

E.g. Find the reference angle for the given angle.

(a) 207° $\boxed{\theta' = 207^\circ - 180^\circ = 27^\circ}$



(b) 1130°. Coterminal angle $= \theta = 1130^\circ - 3 \cdot 360^\circ = 50^\circ$

$\boxed{\theta' = 50^\circ}$

Process to find trig function values of any non quadrantal angle.

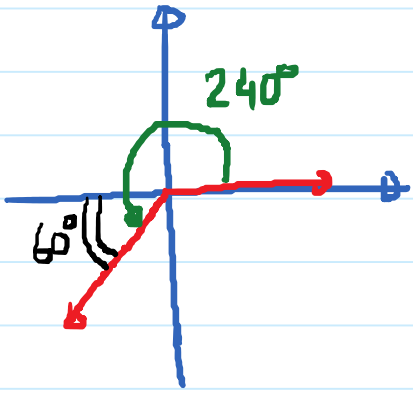
Step 1: If the angle is NOT in $[0^\circ, 360^\circ]$, find the coterminal angle in $[0^\circ, 360^\circ]$

Step 2: Find the reference angle θ'

Step 3: θ' is an acute angle → use SOHCAHTOA or special triangles to calculate the trig function values of it

Step 4: Determine the correct signs for the values in Step 3.

E.g. Find the trig function values of $240°$



Reference angle $= \theta' = 60°$.

$\sin 60° = \frac{\sqrt{3}}{2}$ $\cos 60° = \frac{1}{2}$ $\tan 60° = \sqrt{3}$

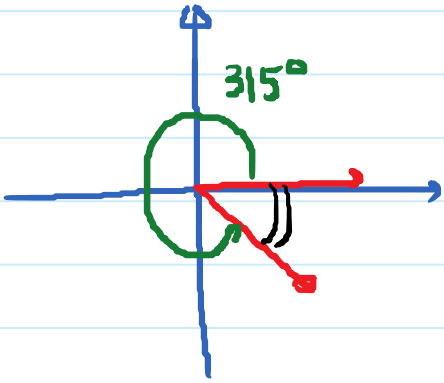
$\csc 60° = \frac{2}{\sqrt{3}}$ $\sec 60° = 2$ $\cot 60° = \frac{1}{\sqrt{3}}$

$\sin 240° = -\frac{\sqrt{3}}{2}$; $\cos 240° = -\frac{1}{2}$; $\tan 240° = \sqrt{3}$

$\csc 240° = -\frac{2}{\sqrt{3}}$; $\sec 240° = -2$; $\cot 240° = \frac{1}{\sqrt{3}}$

E.g. Find the trig function values of $1035°$

Coterminal: $1035° - 2 \cdot 360° = 315°$



Reference angle $= 45°$

$\sin 45° = \frac{1}{\sqrt{2}}$; $\cos 45° = \frac{1}{\sqrt{2}}$ $\tan 45° = 1$

$\csc 45° = \sqrt{2}$; $\sec 45° = \sqrt{2}$; $\cot 45° = 1$

$\sin 1035^\circ = -\frac{1}{\sqrt{2}}$; $\cos 1035^\circ = \frac{1}{\sqrt{2}}$ $\tan 1035^\circ = -1$

$\csc 1035^\circ = -\sqrt{2}$; $\sec 1035^\circ = \sqrt{2}$ $\cot 1035^\circ = -1$

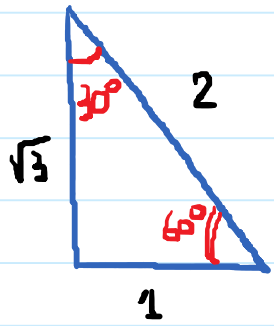
E.g. Find trig function values of -300°.

Coterminal angle: $-300^\circ + 360^\circ = 60^\circ$

Reference angle $= 60^\circ$

$\sin 60^\circ = \frac{\sqrt{3}}{2}$; $\cos 60^\circ = \frac{1}{2}$; $\tan 60^\circ = \sqrt{3}$

$\csc 60^\circ = \frac{2}{\sqrt{3}}$; $\sec 60^\circ = 2$; $\cot 60^\circ = \frac{1}{\sqrt{3}}$

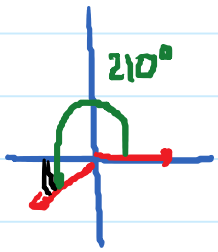


$\sin(-300^\circ) = \frac{\sqrt{3}}{2}$; $\cos(-300^\circ) = \frac{1}{2}$; $\tan(-300^\circ) = \sqrt{3}$

$\csc(-300^\circ) = \frac{2}{\sqrt{3}}$; $\sec(-300^\circ) = 2$; $\cot(-300^\circ) = \frac{1}{\sqrt{3}}$

E.g. Do it for -150°.

Coterminal: $-150^\circ + 360^\circ = 210^\circ$

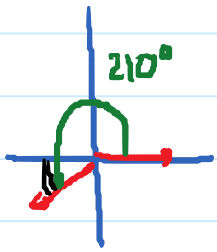


Reference $= 30^\circ$

$\sin 210^\circ = -\frac{1}{2}$

$\cos 210^\circ = -\frac{\sqrt{3}}{2}$

$\tan 210^\circ = \frac{1}{\sqrt{3}}$.



Reference $= 30°$

$\tan 210° = \frac{1}{\sqrt{3}}$.