3.1. Radian Measure

Wednesday, February 12, 2020 9:33 AM

Convert between degrees and radians.

$$180^\circ = \pi \text{ radians} \quad (\pi \approx 3.14\ldots)$$

$$1^\circ = \frac{\pi}{180} \text{ radians}$$

$$1 \text{ radian} = \frac{180}{\pi} \text{ degrees}$$

Convert: Degrees → Radians

Multiply by $\frac{\pi}{180}$

Radians → Degrees

Multiply by $\frac{180}{\pi}$.

E.g. Convert from degrees to radians

(a) $60^\circ \longrightarrow \frac{\cancel{60}^{1}}{\cancel{180}_{3}} \cdot \pi = \frac{\pi}{3}$ (radians)

(b) $-135^\circ \longrightarrow -\frac{\cancel{135}^{3}}{\cancel{180}_{4}} \cdot \pi = -\frac{3\pi}{4}$ (radians)

E.g. Convert from radians to degrees

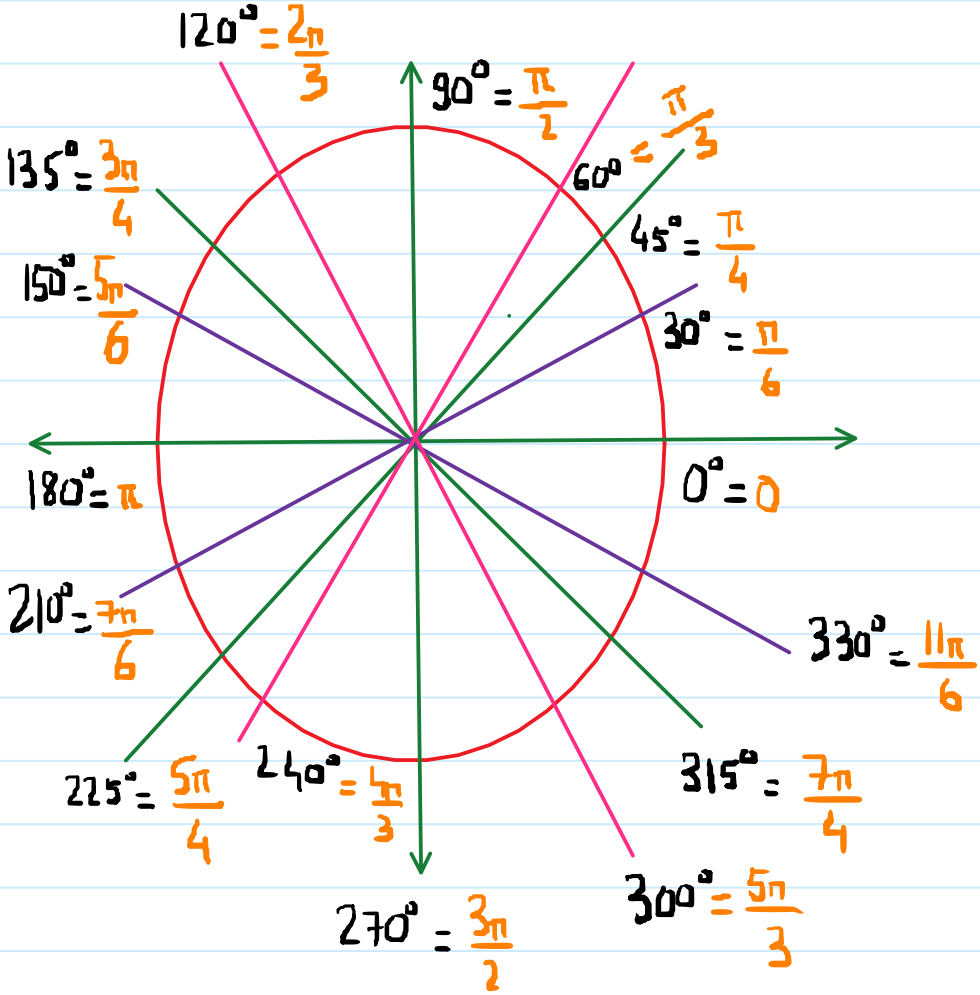
(a) $2.92 \longrightarrow (2.92) \cdot \frac{180}{\pi} \approx 167.3^\circ$

Note: We can substitute 180° for π when we convert from radian to degree.

$\frac{\pi}{6} \longrightarrow 30^\circ$

$\frac{13\pi}{3} \longrightarrow \frac{13 \cdot 180}{3} = 780^\circ$

Important degree measures and their radian equivalences.



Reference angle for the radian measures shown in this picture:

$\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \rightarrow$ Reference angle $= \boxed{\frac{\pi}{6}}$

$\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \rightarrow$ Reference angle $= \boxed{\frac{\pi}{4}}$

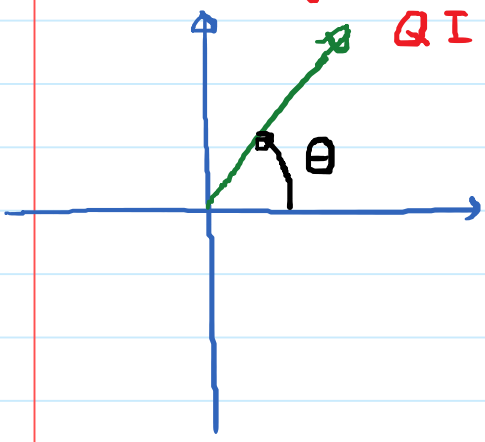
$\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \rightarrow$ Reference angle $= \boxed{\frac{\pi}{3}}$

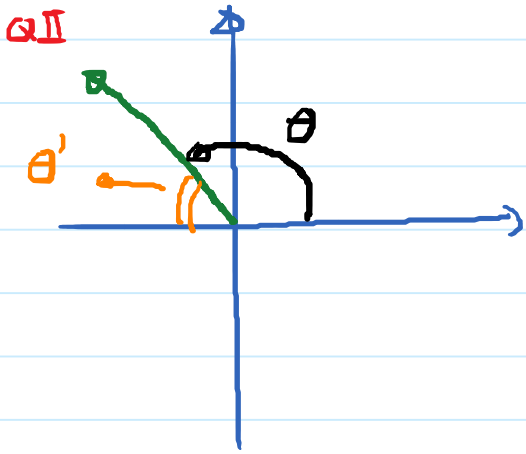
θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$	$\sqrt{3}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2	$\frac{1}{\sqrt{3}}$

E.g. $\sin\frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$; $\tan\frac{5\pi}{4} = 1$; $\sec\frac{5\pi}{6} = -\frac{2}{\sqrt{3}}$.

Reference angle in Radians

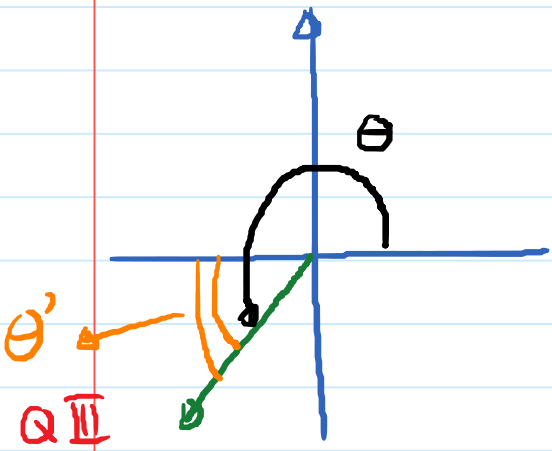
θ is in $[0, 2\pi]$

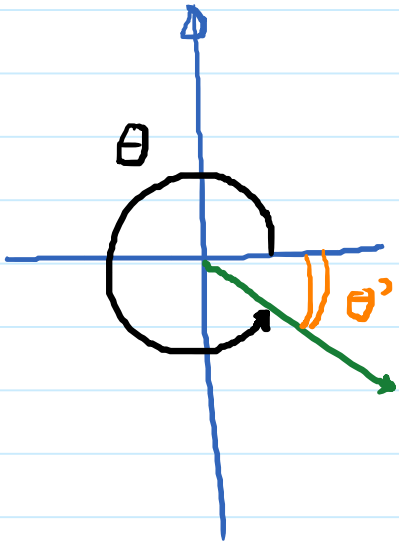




Reference angle: $\theta' = \theta$

$\theta' = \pi - \theta$





$\theta' = \theta - \pi$

$\theta' = 2\pi - \theta$.

E.g. $\theta = 2.1$ radians.

Find reference angle.

$\theta' = \pi - 2.1 \simeq 1.04$