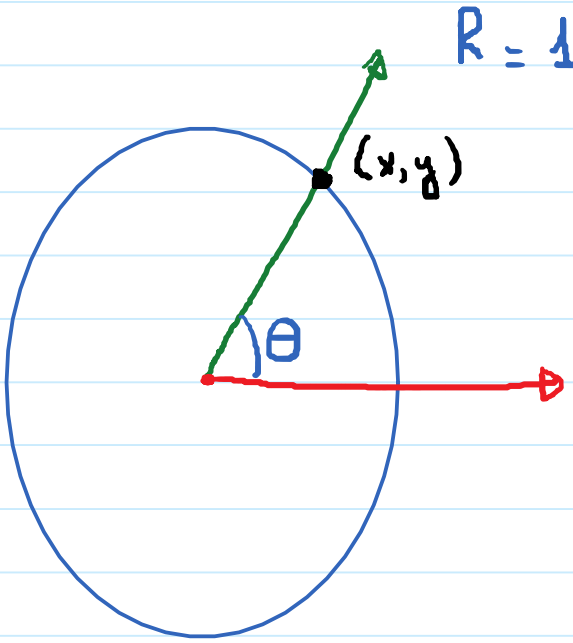
# Unit Circle and Circular Functions

Monday, February 17, 2020 10:28 AM

**Unit Circle:** Circle centered at the origin and Radius = 1




$$\cos\theta = \frac{x}{R} = x$$

$$\sin\theta = \frac{y}{R} = y$$

$$\tan\theta = \frac{y}{x}$$

$$\cot\theta = \frac{x}{y}$$

$$\sec\theta = \frac{1}{x}$$

$$\csc\theta = \frac{1}{y}$$

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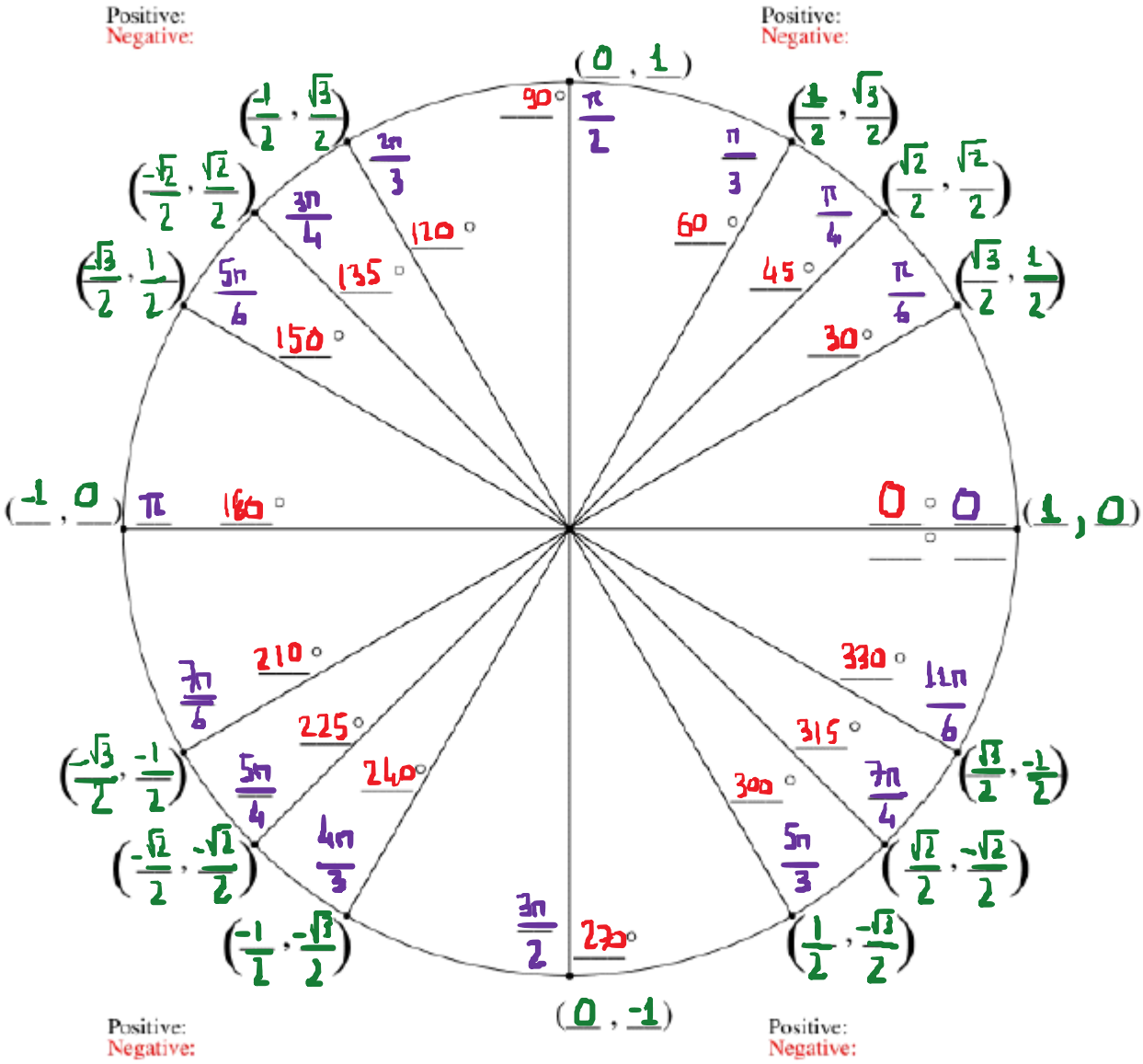


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E.g. $\tan\frac{11\pi}{6} = \frac{\sin\frac{11\pi}{6}}{\cos\frac{11\pi}{6}} = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{2}\cdot\frac{2}{\sqrt{3}} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

$\csc\frac{7\pi}{4} = \frac{1}{\sin\frac{7\pi}{4}} = \frac{1}{-\frac{\sqrt{2}}{2}} = -1\cdot\frac{2}{\sqrt{2}} = -\frac{2}{\sqrt{2}} = \boxed{-\sqrt{2}}$.

$$\sin\left(\frac{13\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

coterminal

$$\frac{13\pi}{3} - \frac{2\pi \cdot 3}{1 \cdot 3} = \frac{13\pi}{3} - \frac{6\pi}{3} = \frac{7\pi}{3}$$

$$\frac{7\pi}{3} - \frac{2\pi \cdot 3}{1 \cdot 3} = \frac{7\pi}{3} - \frac{6\pi}{3} = \frac{\pi}{3}$$

$$\cos\left(-\frac{3\pi}{2}\right) = 0$$

$$-\frac{3\pi}{2} + \frac{2\pi \cdot 2}{1 \cdot 2} = -\frac{3\pi}{2} + \frac{4\pi}{2} = \frac{\pi}{2}$$

coterminal

E.g. Find the exact value of $s$ in the given interval that has the given circular function value:

(a) $\cos(s) = -\frac{1}{2}$ ; $s$ is in $\left[\frac{\pi}{2}, \pi\right]$

$s = \frac{2\pi}{3}$

(b) $\tan(s) = \frac{\sqrt{3}}{3}$ ; $s$ is in $\left[\pi, \frac{3\pi}{2}\right]$

$s = \frac{7\pi}{6}$

(c) $\sin\theta = -\frac{\sqrt{3}}{2}$ ; $\theta$ is in $[0, 2\pi]$

$\theta = \frac{4\pi}{3}, \quad \theta = \frac{5\pi}{3}$

(d) $\cos^2\theta = \frac{1}{2}$ ; $\theta$ in $[0, 2\pi)$

$\cos\theta = \pm\sqrt{\frac{1}{2}} = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

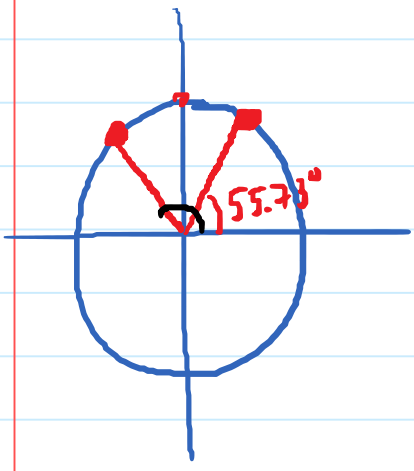
(e) $\tan^2\theta = 3$ , $\theta$ in $[0, 2\pi)$

$\tan\theta = \pm\sqrt{3}$

$\theta = \frac{\pi}{3}, \frac{4\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{3}$.

E.g. Find $s$ in radians.

(a) $\sin s = 0.82639$ ; $s$ in $[0, 2\pi)$



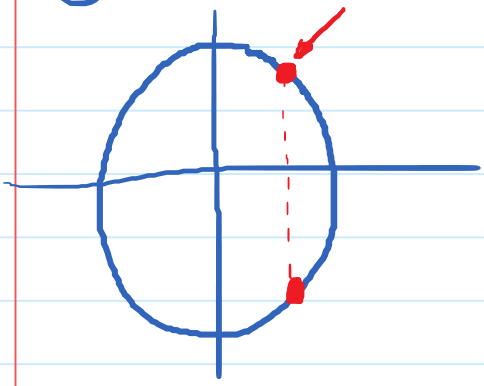

$s = 55.73^\circ$

$s = 180^\circ - 55.73^\circ = 124.27^\circ$ || In degrees

$s = 0.97$.

$s = \pi - 0.97 \approx 2.17$ || In radians

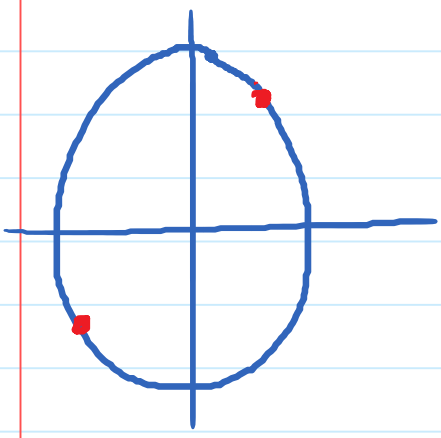
(b) $\cos(s) = 0.42378$ ; $s$ in $[0, 2\pi)$



$s = 1.13$

$s = 2\pi - 1.13 = 5.15$.

(c) $\tan(s) = 2.75$ ; $s$ in $[0, 2\pi)$



$s = 1.22$

$s = \pi + 1.22 = 3.36$.