Section 2.7. Inverse Functions

Tuesday, October 8, 2019 9:36 AM

Obj 1: Verify Inverse Functions

Recall: If f and g are functions, then

$f(g(x))$ means you plug $g(x)$ into f

$g(f(x))$ means you plug $f(x)$ into g.

If after you simplify, both of these expressions equal to x, then f and g are inverse functions of each other.

Definition of the inverse of a function:

Given a function f. If g is a function such that

$$\boxed{f(g(x)) = x \quad \text{and} \quad g(f(x)) = x}$$

then the function g is the inverse of the function f.

We denote the inverse of the function f by f^{-1}

(read as f inverse)

E.g. Verify Inverse Functions.

Given $f(x) = \boxed{3x+2}$; $g(x) = \boxed{\frac{x-2}{3}}$

Q: Verify that each function is the inverse of the other.

Thursday, October 10, 2019 9:45 AM

$$f(g(x)) = f\left(\frac{x-2}{3}\right) = 3\cdot\left(\frac{x-2}{3}\right) + 2$$

$$= \frac{\cancel{3}(x-2)}{\cancel{3}} + 2 = x - 2 + 2 = x$$

So, $\boxed{f(g(x)) = x}$

$$g(f(x)) = g(3x+2) = \frac{(3x \cancel{+2}) \cancel{-2}}{3}$$

$$= \frac{\cancel{3}x}{\cancel{3}} = x$$

So, $\boxed{g(f(x)) = x}$

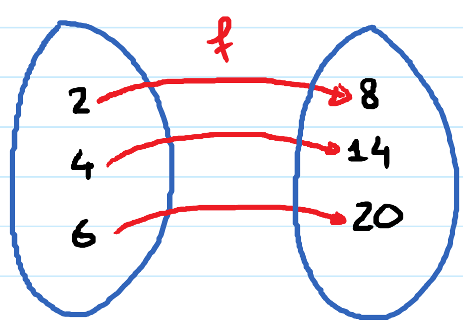
So, f and g are inverse functions of each other.

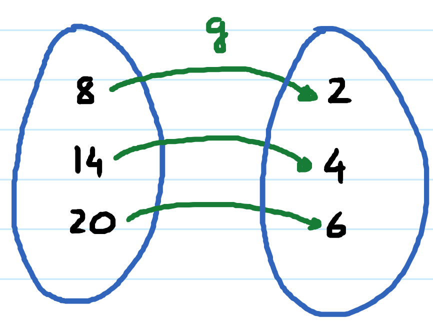
Note: $f(x) = 3x + 2$

x	$f(x)$
2	8
4	14
6	20

$g(x) = \dfrac{x-2}{3}$

x	$g(x)$
8	2
14	4
20	6





$$\underbrace{g(f}_{\text{like } g \text{ cancels } f}(\boxed{2})) = g(8) = \boxed{2}$$

like g cancels f

$$\underbrace{f(g}_{\text{like } f \text{ cancels } g}(\boxed{8})) = f(2) = \boxed{8}$$

like f cancels g

E.g. Given $f(x) = \boxed{\dfrac{2}{x-5}}$; $g(x) = \boxed{\dfrac{2}{x} + 5}$

Verify that each function is the inverse of the other.

Sol:

$$f(g(x)) = f\left(\frac{2}{x} + 5\right) = \frac{2}{\left(\frac{2}{x} \cancel{+5}\right) \cancel{-5}}$$

$$= \frac{2}{\frac{2}{x}} = \frac{\frac{2}{1}}{\frac{2}{x}} = \frac{2}{1} \cdot \frac{x}{2} = \frac{\cancel{2}x}{\cancel{2}} = x$$

$$g(f(x)) = g\left(\frac{2}{x-5}\right) = \frac{2}{\left(\frac{2}{x-5}\right)} + 5$$

$$= \frac{2}{1} \cdot \frac{x-5}{2} + 5$$

$$= \frac{\cancel{2}(x-5)}{\cancel{2}} + 5 = x \cancel{-5} \cancel{+5} = x$$

Obj 2: Find the inverse of a function.

E.g. Find the inverse of $f(x) = 2x + 7$

Process:

Step 1: Replace the notation $f(x)$ with y

$$\boxed{y} = 2\boxed{x} + 7$$

Step 2: Interchange x and y in the above equation

$$x = 2y + 7$$

Step 3: Solve for y by itself in the equation in Step 2

$$x = 2y + 7$$

$x - 7 = 2y$ (Subtract 7 from both sides)

$\dfrac{x-7}{2} = y$ (Divide both sides by 2)

$$y = \frac{x-7}{2}$$

Step 4: Replace y by the notation $f^{-1}(x)$

$$\boxed{f^{-1}(x) = \frac{x-7}{2}}$$

Answer. This is the inverse function of f.

E.g. Find the inverse of $f(x) = 4x^3 - 1$.

Step 1: Replace $f(x)$ by y

$$y = 4x^3 - 1$$

Step 2: Interchange x and y

$$x = 4y^3 - 1$$

Step 3: Solve for y by itself.

$$x + 1 = 4y^3$$

$$\frac{x+1}{4} = y^3$$

$\sqrt[3]{\dfrac{x+1}{4}} = y$ (Take cube root of both sides)

$$y = \sqrt[3]{\frac{x+1}{4}}$$

Step 4: Replace y with $f^{-1}(x)$

$$\boxed{f^{-1}(x) = \sqrt[3]{\frac{x+1}{4}}} \longleftarrow \text{Answer}$$

E.g. Find the inverse of $f(x) = \dfrac{5}{7x-1}$

Step 1: Replace $f(x)$ with y

$$y = \frac{5}{7x-1}$$

Step 2: Interchange x and y

$$x = \frac{5}{7y-1}$$

Step 3: Solve for y by itself

Multiply both sides by the denominator:

$$x(7y-1) = \frac{5}{\cancel{(7y-1)}}\cancel{(7y-1)}$$

$$x(7y-1) = 5$$

$$7xy - x = 5$$

$$7xy = 5 + x$$

$$y = \frac{5+x}{7x}$$

Step 4: Replace y by $f^{-1}(x)$

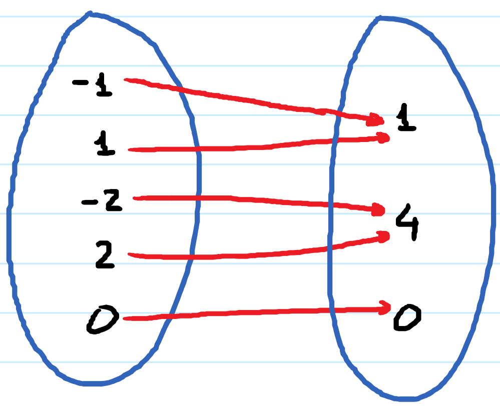
$$\boxed{f^{-1}(x) = \frac{5+x}{7x}}$$

Obj 3: The Horizontal line Test and One-to-one functions

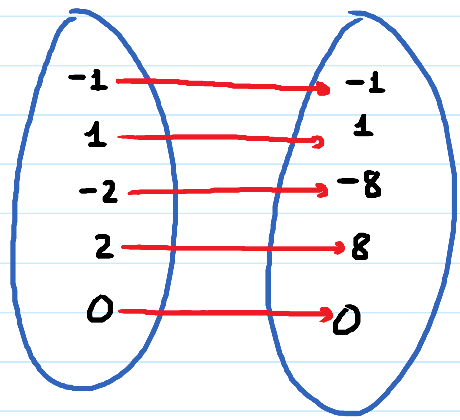
Note: Not all functions have inverse functions.

Only one-to-one functions have inverse functions.

E.g.



This is NOT a one-to-one function



This is a one-to-one function

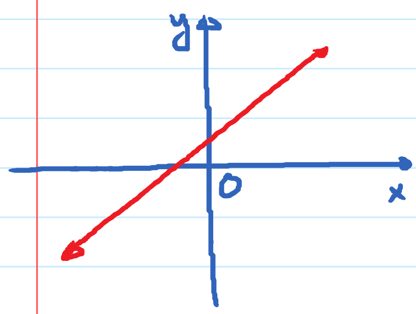
To be a function: each x has only one y.

To be a one-to-one function: it needs to be a function first and each y has only one x.

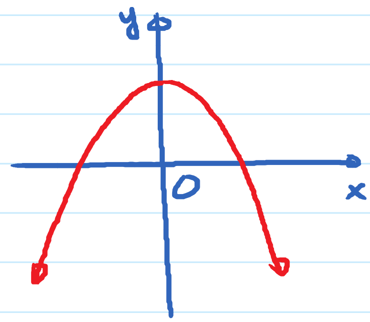
Horizontal line Test

A function is one-to-one; hence, has an inverse if there is NO horizontal line that intersects the graph at more than one point.

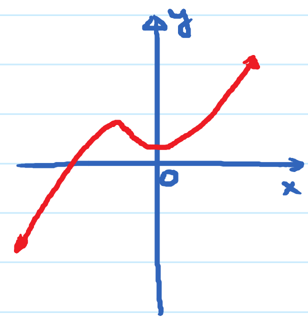
E.g.



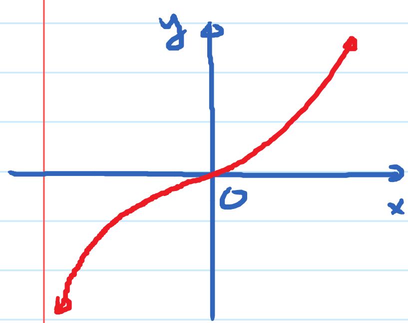
one-to-one
(has inverse)



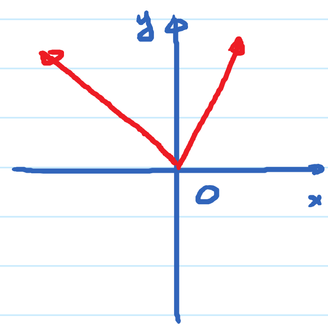
Not one-to-one



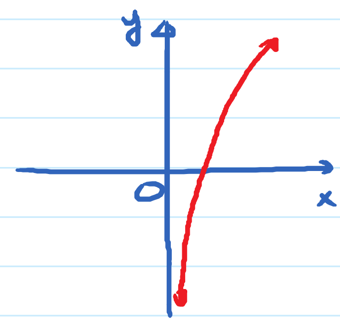
Not one-to-one



one-to-one
(has inverse)



not one-to-one



one-to-one
(has inverse)