4.2 Graph transformations of sine and cosine.

Graphs of the form

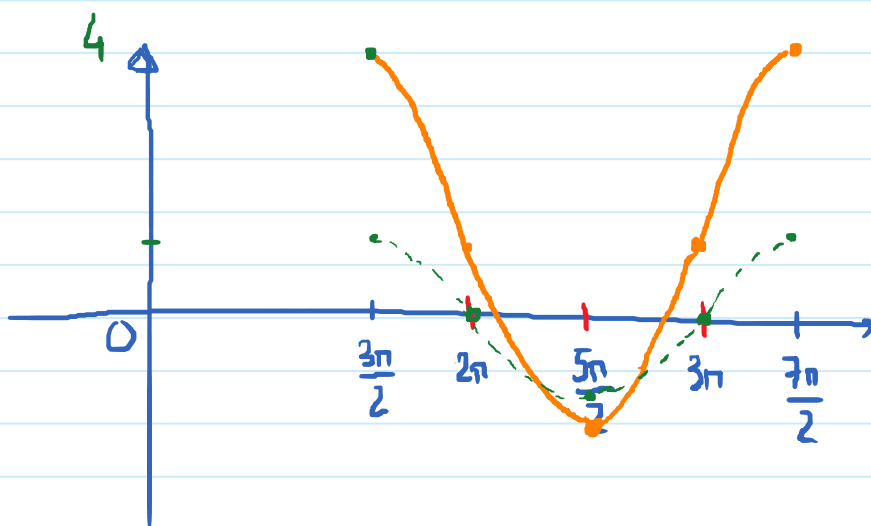$$y = a \sin(bx + c) + d$$

$$\text{or} \quad y = a \cos(bx + c) + d$$

E.g. $y = 3 \cos\left(x - \frac{3\pi}{2}\right) + 1$

$x - \frac{3\pi}{2} = 0$

$$\boxed{x = \frac{3\pi}{2}}$$

$x - \frac{3\pi}{2} = 2\pi$

$x = \frac{2\pi \cdot 2}{1 \cdot 2} + \frac{3\pi}{2} = \frac{4\pi}{2} + \frac{3\pi}{2} = \frac{7\pi}{2}$.

$$y = A \sin(Bx \pm C) + D \quad \text{or} \quad y = A \cos(Bx \pm C) + D$$

$$(B > 0, \ C > 0)$$

$(B > 0, C > 0)$

Amplitude: $|A|$ Period: $\dfrac{2\pi}{B}$

Phase Shift: $\dfrac{C}{B}$ to left if $\oplus$

                 $\dfrac{C}{B}$ to right if $\ominus$

E.g. $y = -2\sin\left(\boxed{x + \dfrac{\pi}{4}}\right) - 3$.

(a) Find amplitude, period, phase shift

(b) Graph in 1-cycle.

Sol:

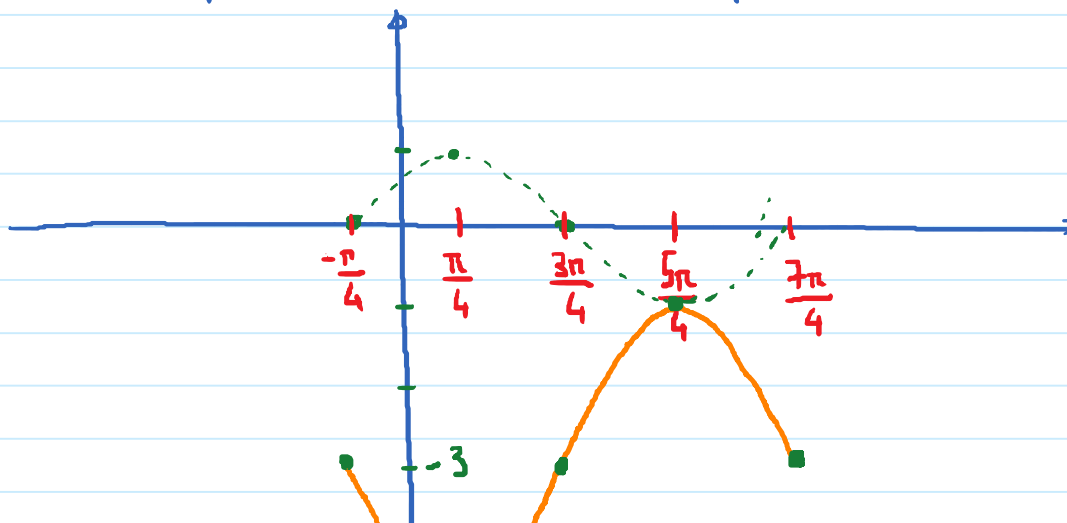(a) $A = -2$.   Amplitude $= |-2| = 2$

    $B = 1$ Period $= \dfrac{2\pi}{1} = 2\pi$.
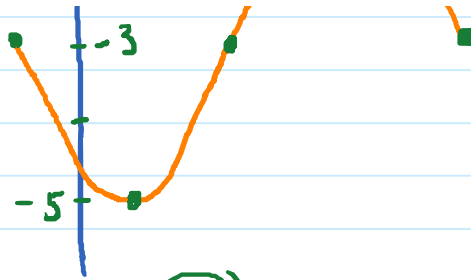
    $C = \dfrac{\pi}{4}$ Phase shift $= \dfrac{\pi}{4}$ to left.

(b) $x + \dfrac{\pi}{4} = 0$; $x + \dfrac{\pi}{4} = 2\pi$

     $x = -\dfrac{\pi}{4}$ $x = \dfrac{7\pi}{4}$

E.g. $y = \cos\left(\frac{1}{2}x - \frac{\pi}{4}\right) + 1$

(a) Amplitude, Phase shift, Period

(b) Graph in 1-cycle.

(a) $A = 1$ ∴ Amplitude $= |A| = 1$; Period $= \frac{2\pi}{\frac{1}{2}} = 4\pi$.

Phase shift $= \frac{\frac{\pi}{4}}{\frac{1}{2}} = \frac{\pi}{4} \cdot \frac{2}{1} = \frac{\pi}{2}$ to right.

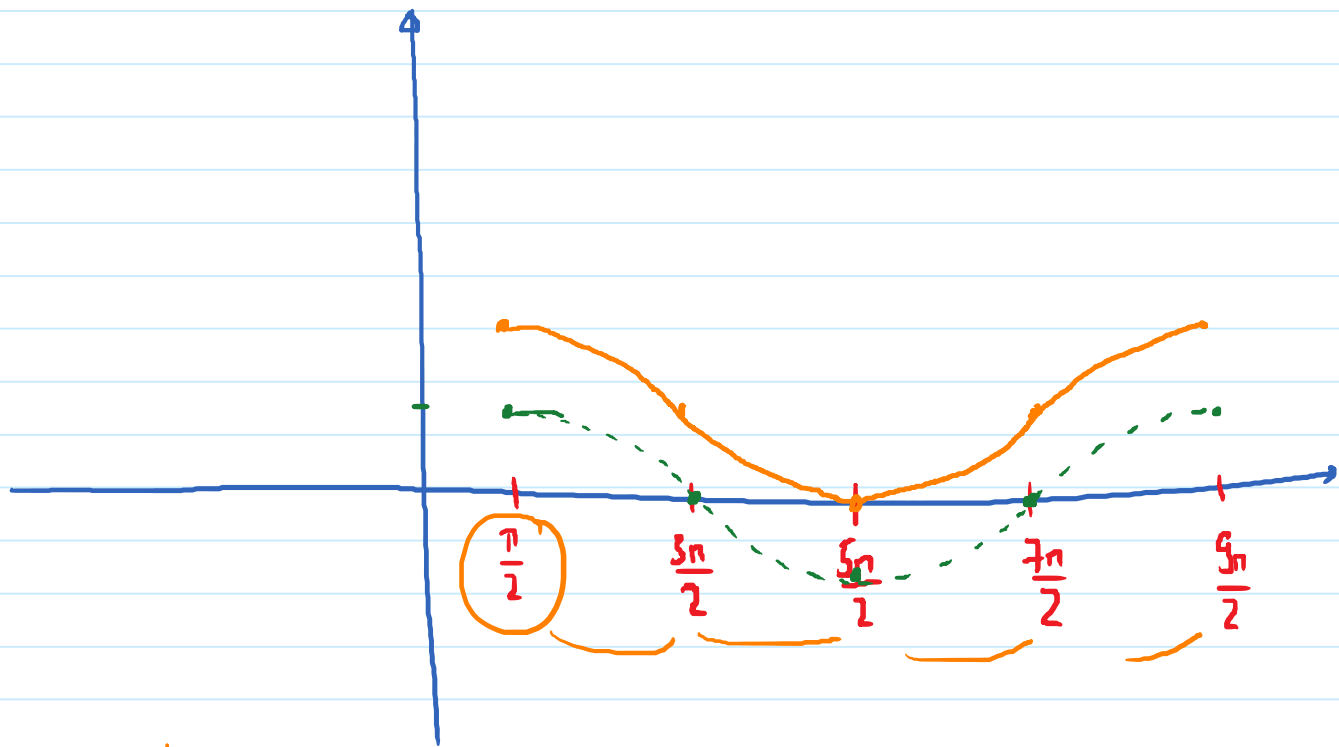(b) $\frac{1}{2}x - \frac{\pi}{4} = 0$; $\frac{1}{2}x - \frac{\pi}{4} = 2\pi$

$\frac{1}{2}x = \frac{\pi}{4}$ $\frac{1}{2}x = \frac{2\pi \cdot 4}{1 \cdot 4} + \frac{\pi}{4} = \frac{9\pi}{4}$

$\boxed{x = \frac{\pi}{2}}$ $\boxed{x = \frac{9\pi}{2}}$

$$\frac{\pi}{2} + \frac{\pi \cdot 2}{1 \cdot 2} = \frac{\pi}{2} + \frac{2\pi}{2}$$

Period = 4π

1 quarter of period = $\dfrac{4\pi}{4} = \pi$