

Student: _____
Date: _____

Course: College Algebra

Test 3 covers 3.1, 3.2, 3.3, 3.4 and 3.5

1. In the following exercise, find the coordinates of the vertex for the parabola defined by the given quadratic function.

$$f(x) = 3x^2 + 6x + 9$$

The vertex is _____. (Type an ordered pair.)

2. Consider the function $f(x) = 3x^2 - 18x - 1$.

- Determine, without graphing, whether the function has a minimum value or a maximum value.
- Find the minimum or maximum value and determine where it occurs.
- Identify the function's domain and its range.

a. The function has a (1) _____ value.

b. The minimum/maximum value is _____. It occurs at $x =$ _____.

c. The domain of f is _____. (Type your answer in interval notation.)

The range of f is _____. (Type your answer in interval notation.)

- (1) ☐ maximum
☐ minimum

3. Consider the function $f(x) = -3x^2 + 18x - 9$.

- Determine, without graphing, whether the function has a minimum value or a maximum value.
- Find the minimum or maximum value and determine where it occurs.
- Identify the function's domain and its range.

a. The function has a (1) _____ value.

b. The minimum/maximum value is _____. It occurs at $x =$ _____.

c. The domain of f is _____. (Type your answer in interval notation.)

The range of f is _____. (Type your answer in interval notation.)

- (1) ☐ maximum
☐ minimum

4. Fill in each blank so that the resulting statement is true.

Consider the quadratic function $f(x) = ax^2 + bx + c$, $a \neq 0$. If $a > 0$, then f has a minimum that occurs at $x = \underline{\hspace{2cm}}$. This minimum value is $\underline{\hspace{2cm}}$. If $a < 0$, then f has a maximum that occurs at $x = \underline{\hspace{2cm}}$. This maximum value is $\underline{\hspace{2cm}}$.

Consider the quadratic function $f(x) = ax^2 + bx + c$, $a \neq 0$. If $a > 0$, then f has a minimum that occurs at

$x =$ (1) $\underline{\hspace{2cm}}$ This minimum value is (2) $\underline{\hspace{2cm}}$ If $a < 0$, then f has a maximum that occurs at

$x =$ (3) $\underline{\hspace{2cm}}$ This maximum value is (4) $\underline{\hspace{2cm}}$

- | | | | |
|---|--|---|---|
| (1) <input type="radio"/> $f\left(-\frac{b}{2a}\right)$. | (2) <input type="radio"/> $f\left(\frac{2a}{b}\right)$. | (3) <input type="radio"/> $\frac{2a}{b}$. | (4) <input type="radio"/> $f\left(-\frac{b}{2a}\right)$. |
| <input type="radio"/> $-\frac{b}{2a}$. | <input type="radio"/> $f\left(-\frac{b}{2a}\right)$. | <input type="radio"/> $f\left(\frac{2a}{b}\right)$. | <input type="radio"/> $-\frac{b}{2a}$. |
| <input type="radio"/> $f\left(\frac{2a}{b}\right)$. | <input type="radio"/> $-\frac{b}{2a}$. | <input type="radio"/> $f\left(-\frac{b}{2a}\right)$. | <input type="radio"/> $f\left(\frac{2a}{b}\right)$. |
| <input type="radio"/> $\frac{2a}{b}$. | <input type="radio"/> $\frac{2a}{b}$. | <input type="radio"/> $-\frac{b}{2a}$. | <input type="radio"/> $\frac{2a}{b}$. |

5. Determine whether the function is a polynomial function. If it is, identify the degree.

$$f(x) = 3x^6 + 5x^7$$

Choose the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☐ A. It is a polynomial. The degree of the polynomial is $\underline{\hspace{2cm}}$.
- ☐ B. It is not a polynomial.

6. Use the leading coefficient test to determine the end behavior of the graph of the given polynomial function.

$$f(x) = 6x^7 - 2x^6 + 2x + 5$$

- ☐ A. Falls left & rises right.
- ☐ B. Rises left & rises right.
- ☐ C. Falls left & falls right.
- ☐ D. Rises left & falls right.
- ☐ E. None of the above.

7. Find the zeros for the polynomial function and give the multiplicity for each zero. State whether the graph crosses the x-axis or touches the x-axis and turns around at each zero.

$$f(x) = -7(x - 3)(x - 4)^2$$

Determine the zero(s).

The zero(s) is/are _____.

(Type integers or decimals. Use a comma to separate answers as needed.)

Determine the multiplicities of the zero(s). Select the correct choice below and, if necessary, fill in the answer box(es) within your choice.

- ☐ A. There are two zeros. The multiplicity of the smallest zero is _____. The multiplicity of the largest zero is _____.
(Simplify your answers.)
- ☐ B. There is one zero. The multiplicity of the zero is _____.
(Simplify your answer.)
- ☐ C. There are three zeros. The multiplicity of the smallest zero is _____. The multiplicity of the largest zero is _____. The multiplicity of the other zero is _____.
(Simplify your answers.)

Determine the behavior of the function at each zero. Select the correct choice below and, if necessary, fill in the answer boxes within your choice.

- ☐ A. The graph crosses the x-axis at $x =$ _____ and touches the x-axis and turns around at $x =$ _____.
(Simplify your answers. Type integers or decimals. Use a comma to separate answers as needed.)
- ☐ B. The graph crosses the x-axis at all zeros.
- ☐ C. The graph touches the x-axis and turns around at all zeros.
-

8. Use the given function to complete parts (a) through (e) below.

$$f(x) = x^4 - 9x^2$$

a) Use the Leading Coefficient Test to determine the graph's end behavior.

- ☐ A. The graph of $f(x)$ falls left and rises right.
☐ B. The graph of $f(x)$ falls left and falls right.
☐ C. The graph of $f(x)$ rises left and falls right.
☐ D. The graph of $f(x)$ rises left and rises right.

b) Find the x-intercepts.

x = _____

(Type an integer or a decimal. Use a comma to separate answers as needed.)

At which zeros does the graph of the function cross the x-axis? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☐ A. x = _____ (Type an integer or a decimal. Use a comma to separate answers as needed.)
☐ B. There are no x-intercepts at which the graph crosses the x-axis.

At which zeros does the graph of the function touch the x-axis and turn around? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☐ A. x = _____ (Type an integer or a decimal. Use a comma to separate answers as needed.)
☐ B. There are no x-intercepts at which the graph touches the x-axis and turns around.

c) Find the y-intercept by computing $f(0)$.

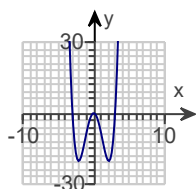
$f(0) =$ _____

d) Determine the symmetry of the graph.

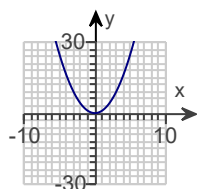
- ☐ Odd; origin symmetry
☐ Even; y-axis symmetry
☐ Neither

e) Determine the graph of the function.

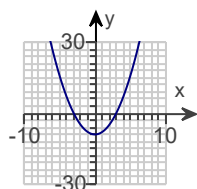
☐ A.



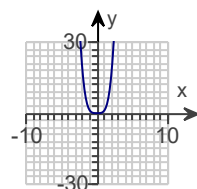
☐ B.



☐ C.



☐ D.



9. Fill in the blank so that the resulting statement is true.

To divide $x^3 + 5x^2 - 4x + 1$ by $x - 3$ using synthetic division, the first step is to write

To divide $x^3 + 5x^2 - 4x + 1$ by $x - 3$ using synthetic division, the first step is to write

10. Divide using synthetic division.

$$(x^3 + 9x^2 - 4x + 8) \div (x - 5)$$

$$(x^3 + 9x^2 - 4x + 8) \div (x - 5) = \underline{\hspace{2cm}} + \frac{\hspace{2cm}}{x - 5}$$

(Simplify your answers. Do not factor. Use integers or fractions for any numbers in the expressions.)

11. Divide using synthetic division.

$$\frac{x^5 + 3x^3 - 4}{x - 1}$$

$$\frac{x^5 + 3x^3 - 4}{x - 1} = \underline{\hspace{2cm}} + \frac{\hspace{2cm}}{x - 1}$$

(Simplify your answers. Do not factor.)

12. Use synthetic division and the remainder theorem to find the indicated function value.

$$f(x) = 4x^3 - 2x^2 - 5x + 5; f(-4)$$

$$f(-4) =$$

13. Fill in the blanks so that the resulting statement is true.

Consider solving $4x^3 + 23x^2 - 7x - 6 = 0$. The synthetic division shown below indicates that (1) _____ is a root.

$$\begin{array}{rrrr} -6 & 4 & 23 & -7 & -6 \\ & & -24 & 6 & 6 \\ \hline & 4 & -1 & -1 & 0 \end{array}$$

Based on the synthetic division, $4x^3 + 23x^2 - 7x - 6 = 0$ can be written in factored form as (2) _____

- (1) $\begin{array}{c} \bigcirc -6 \\ \bigcirc 6 \end{array}$ (2) $\begin{array}{l} \bigcirc (x+6)(4x^2 - x - 1) = 0. \\ \bigcirc (x-6)(4x^2 + x + 1) = 0. \\ \bigcirc (x+6)(4x^2 + x - 1) = 0. \\ \bigcirc (x-6)(4x^2 - x - 1) = 0. \end{array}$

14. Use the Rational Zero Theorem to list all possible rational zeros for the given function.

$$f(x) = 4x^4 - x^3 + 3x^2 - 2x - 22$$

Which of the following is the complete list of possible zeros of the given function?

- ☐ A. $\pm 1, \pm 2, \pm 11, \pm \frac{1}{2}, \pm \frac{11}{2}$
- ☐ B. $\pm 1, \pm 2, \pm 11, \pm 22, \pm \frac{1}{2}, \pm \frac{11}{2}, \pm \frac{1}{4}, \pm \frac{11}{4}$
- ☐ C. $\pm 1, \pm 2, \pm \frac{11}{2}, \pm \frac{1}{4}, \pm \frac{11}{4}$
- ☐ D. The function has no rational zeros.

- *15. Find a polynomial function of degree 3 with real coefficients that has the given zeros.

$$-1, 2, -4$$

The polynomial function is $f(x) = x^3 + \underline{\hspace{2cm}}x^2 - 6x - 8$.

- *16. The polynomial function $f(x)$ has the given zero. Find the other zeros.

$$f(x) = x^3 + 4x^2 - 6x - 24; -4$$

The other zeros are .

(Type exact answers, using radicals as needed. Use a comma to separate answers as needed.)

17. Find the domain of the following rational function.

$$H(x) = \frac{-9x^2}{(x-8)(x+6)}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☐ A. The domain of $H(x)$ is $\{x|x \neq \underline{\hspace{2cm}}\}$. (Type an integer or a fraction. Use a comma to separate answers as needed.)
- ☐ B. The domain of $H(x)$ has no restrictions.

18. Find the vertical asymptotes, if any, and the values of x corresponding to holes, if any, of the graph of the rational function.

$$f(x) = \frac{x - 8}{x^2 - 13x + 40}$$

Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice. (Type an integer or a fraction. Use a comma to separate answers as needed.)

- ☐ A. Vertical asymptote(s) at $x =$ _____ and hole(s) at $x =$ _____
- ☐ B. Vertical asymptote(s) at $x =$ _____
- ☐ C. Hole(s) at $x =$ _____
- ☐ D. There are no discontinuities.

-
19. Find the horizontal asymptote, if any, of the graph of the rational function.

$$g(x) = \frac{10x^2}{5x^2 + 4}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☐ A. The horizontal asymptote is _____. (Type an equation.)
- ☐ B. There is no horizontal asymptote.

-
20. Find the horizontal asymptote, if any, of the graph of the rational function.

$$h(x) = \frac{14x^3}{7x^2 + 6}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☐ A. The horizontal asymptote is _____. (Type an equation.)
- ☐ B. There is no horizontal asymptote.

1. $(-1, 6)$

2. (1) minimum

-28

3

$(-\infty, \infty)$

$[-28, \infty)$

3. (1) maximum

18

3

$(-\infty, \infty)$

$(-\infty, 18]$

4. (1) $-\frac{b}{2a}$.

(2) $f\left(-\frac{b}{2a}\right)$.

(3) $-\frac{b}{2a}$.

(4) $f\left(-\frac{b}{2a}\right)$.

5. A. It is a polynomial. The degree of the polynomial is 7.

6. A. Falls left & rises right.

7. 3,4

A.

There are two zeros. The multiplicity of the smallest zero is 1. The multiplicity of the largest zero is 2.

(Simplify your answers.)

A. The graph crosses the x-axis at $x =$ 3 and touches the x-axis and turns around at $x =$ 4.
(Simplify your answers. Type integers or decimals. Use a comma to separate answers as needed.)

8. D. The graph of $f(x)$ rises left and rises right.

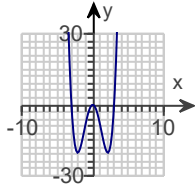
$-3, 3, 0$

A. $x =$ $-3, 3$ (Type an integer or a decimal. Use a comma to separate answers as needed.)

A. $x =$ 0 (Type an integer or a decimal. Use a comma to separate answers as needed.)

0

Even; y-axis symmetry



A.

9. 3

1

5

-4

1

10. $x^2 + 14x + 66$

338

11. $x^4 + x^3 + 4x^2 + 4x + 4$

0

12. -263

13. (1) -6

(2) $(x + 6)(4x^2 - x - 1) = 0.$

14. B. $\pm 1, \pm 2, \pm 11, \pm 22, \pm \frac{1}{2}, \pm \frac{11}{2}, \pm \frac{1}{4}, \pm \frac{11}{4}$

15. 3

16. $\sqrt{6}, -\sqrt{6}$

17. A.

The domain of $H(x)$ is $\{x|x \neq \underline{8, -6}\}$. (Type an integer or a fraction. Use a comma to separate answers as needed.)

18. A. Vertical asymptote(s) at $x = \underline{5}$ and hole(s) at $x = \underline{8}$

19. A. The horizontal asymptote is $\underline{y = 2}$. (Type an equation.)

20. B. There is no horizontal asymptote.
