Practice 3 - Solutions

① Vertex Formula : $\left(\boxed{-\frac{b}{2a}}, \boxed{f\left(-\frac{b}{2a}\right)}\right)$

x vertex   y vertex

$a=3$; $b=6$; $c=9$.

$x_{vertex} = -\frac{b}{2a} = -\frac{6}{2\cdot(3)} = \boxed{-1}$

$y_{vertex} = 3\cdot(-1)^2 + 6(-1) + 9 = 3-6+9 = \boxed{6}$

plug x_{vertex} into formula

Vertex is $\boxed{(-1, 6)}$

② ⓐ $a=3$; $b=-18$; $c=-1$

Since $a > 3$, the function has a $\boxed{\text{minimum}}$ value.

(Parabola is upward ∪ when $a>0$)

ⓑ Minimum Value = y_{vertex}. It occurs at $x = x_{vertex}$.

So, we just need to find the vertex.

$x_{vertex} = -\frac{b}{2a} = -\frac{(-18)}{2(3)} = 3$

$y_{vertex} = 3\cdot(3)^2 - 18(3) - 1 = -28$

So, the minimum value is $\boxed{-28}$. It occurs at $x = \boxed{3}$

ⓒ The domain of f is $\boxed{(-\infty, \infty)}$

The range of f is $\boxed{(-28, \infty)}$

(Range = (y_{vertex}, ∞) if $a>0$.

Range = $(-\infty, y_{vertex})$ if $a<0$.)

③ ⓐ $a=-3$; $b=18$, $c=-9$.

Since $a<0$, the function has a $\boxed{\text{maximum}}$ value.

(Parabola is downward when $a<0$. ∩)

ⓑ $x_{vertex} = -\frac{b}{2a} = -\frac{18}{2(-3)} = 3$.

$y_{vertex} = -3(3)^2 + 18(3) - 9 = 18$

So, the maximum value is $\boxed{18}$. It occurs at $x = \boxed{3}$

(c) The domain of f is $(-\infty, \infty)$

The range of f is $(-\infty, 18)$ (If $a < 0$, then the range is $(-\infty, y_{vertex})$)

(4) $f(x) = ax^2 + bx + c$.

If $a > 0$, then f has a minimum that occurs at $x = \boxed{-\frac{b}{2a}}$.

This minimum value is $\boxed{f\left(-\frac{b}{2a}\right)}$

If $a < 0$, then f has a maximum that occurs at $x = \boxed{-\frac{b}{2a}}$.

This maximum value is $\boxed{f\left(-\frac{b}{2a}\right)}$

(5) $f(x) = 3x^6 + 5x^7$

It is a polynomial. Degree $= \boxed{7}$

(degree = highest power of x)

(6) $f(x) = 6x^7 - 2x^6 + 2x + 5$.

Degree $= 7$ (odd) . leading coefficient $= 6$ (positive)

So, Falls left, rises right.

(7) To find the zeros, set $f(x) = 0$.

$$-7(x-3)^{\boxed{1}}(x-4)^{\boxed{2}} = 0$$

$\to x - 3 = 0$ or $(x-4)^2 = 0$

$\to x = 3$ or $x - 4 = 0$

$\to x = 4$.

$\underbrace{\quad}$

multiplicity 1

odd

crosses x-axis at $x=3$

$\rightarrow \underbrace{x=4}.$

multiplicity 2

even

touches x-axis and turns around at $x=2$

8) $f(x) = x^4 - 9x^2$

a) Degree = 4 (Even). leading coefficient = 1 (positive)

$\longrightarrow$ Graph rises left and rises right.

b) x-intercepts: Set $f(x)=0$. $x^4 - 9x^2 = 0$

$\rightarrow x^2(x^2-9) = 0 \rightarrow x^2(x-3)(x+3) = 0$

$\rightarrow x^2 = 0$ or $x-3=0$ or $x+3=0$

$\rightarrow x = 0$ (Multiplicity 2) | $x=3$ (Multiplicity 1) | $x=-3$ (Multiplicity 1)

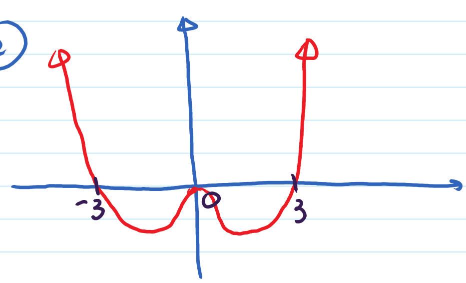
So, at $x = \boxed{3, -3}$: The graph crosses the x-axis. (odd multiplicity)

at $x = \boxed{0}$: The graph touches the x-axis and turns around. (even multiplicity)

c) y-intercept: Put $x=0$ $\rightarrow f(0) = (0)^4 - 9(0)^2 = \boxed{0}$

d) Even; y-axis symmetry.

e)



9) $x^3 + 5x^2 - 4x + 1$ divided by $x-3$.

First step: $3 \,|\; 1 \quad 5 \quad -4 \quad 1$