First step: $\begin{array}{r|rrrr} & 1 & 5 & \overset{0}{-4} & 1 \\ 3 & & & & \\ \hline \end{array}$

(10) $(x^3 + 9x^2 - 4x + 8) \div (x - 5)$

$$\begin{array}{r|rrrr} & 1 & 9 & -4 & 8 \\ 5 & & 5 & 70 & 330 \\ \hline & 1 & 14 & 66 & \boxed{338} \end{array} \longrightarrow \text{Remainder}$$

Quotient: $1x^2 + 14x + 66$

Answer: $(x^3 + 9x^2 - 4x + 8) \div (x - 5) = \underbrace{x^2 + 14x + 66}_{\text{Quotient}} + \dfrac{\overset{\text{Remainder}}{\boxed{338}}}{x - 5}$

(11) $\dfrac{x^5 + 3x^3 - 4}{x - 1}$

$$\begin{array}{r|rrrrrr} & 1 & 0 & 3 & 0 & 0 & -4 \\ 1 & & 1 & 1 & 4 & 4 & 4 \\ \hline & \underbrace{1 \quad 1 \quad 4 \quad 4 \quad 4}_{\text{Quotient}} & & & & & \boxed{0} \end{array} \longrightarrow \text{Remainder}$$

$x^4 + x^3 + 4x^2 + 4x + 4$

Answer: $x^4 + x^3 + 4x^2 + 4x + 4 + \dfrac{0}{x - 1} = \boxed{x^4 + x^3 + 4x^2 + 4x + 4}$

(12) Remainder Theorem: Divide $f(x)$ by $x - a$. Remainder $= f(a)$

So, $\begin{array}{r|rrrr} & 4 & -2 & -5 & 5 \\ -4 & & -16 & 72 & -268 \\ \hline & 4 & -18 & 67 & \boxed{-263} \end{array} \longrightarrow \text{Remainder}$

So, $\boxed{f(-4) = -263}$

(13) Since the remainder is 0, this indicates that $\underbrace{-6 \text{ is a root}}$.

And $4x^3 + 23x^2 - 7x - 6 = 0$ can be factored as:

$(x + 6)\underbrace{(4x^2 - x - 1)}_{\text{Quotient}} = 0$

So $(x + 6)$ is a factor of the given polynomial.

(14) $\dfrac{p}{q}$ $p \rightarrow$ factor of constant term

$q \rightarrow$ factor of leading coefficient

Constant term: $-22 \rightarrow$ Factor: $\pm 1, \pm 2, \pm 11, \pm 22$

leading coefficient: $4 \rightarrow$ Factor: $\pm 1, \pm 2, \pm 4$.

$$\frac{p}{q} = \left\{ \frac{\pm 1, \pm 2, \pm 11, \pm 22}{\pm 1, \pm 2, \pm 4} \right\}$$

$$= \boxed{\left\{ \pm 1, \pm 2, \pm 11, \pm 22, \pm \frac{1}{2}, \pm \frac{11}{2}, \pm \frac{1}{4}, \pm \frac{11}{4} \right\}}$$

(15) $f(x) = (x+1)(x-2)(x+4)$

$= (x^2 - x - 2)(x+4)$

$= x^3 + 4x^2 - x^2 - 4x - 2x - 8$

$= \boxed{x^3 + 3x^2 - 6x - 8.}$

(16) Step 1: Divide:

$$\begin{array}{r|rrrr} -4 & 1 & 4 & -6 & -24 \\ & & -4 & 0 & 24 \\ \hline & 1 & 0 & -6 & 0 \end{array}$$

Quotient $= x^2 - 6$.

Step 2: Set quotient $= 0$ and solve:

$x^2 - 6 = 0 \rightarrow x^2 = 6 \rightarrow x = \boxed{\pm\sqrt{6}}$

(17) To find domain: Set denominator $= 0$.

$(x-8)(x+6) = 0 \rightarrow x - 8 = 0$; $x + 6 = 0$

$\rightarrow x = 8$; $x = -6$.

So, Domain $= \boxed{\{ x \mid x \neq 8, x \neq -6 \}}$

(18) Find vertical asymptote(s)

Step 1: Factor top and bottom (if possible).

Step 1: Factor top and bottom (if possible).

$$f(x) = \frac{x-8}{x^2-13x+40} = \frac{x-8}{(x-8)(x-5)}$$

Step 2: Cancel common factor(s) (if any).

$$f(x) = \frac{\cancel{x-8}}{(\cancel{x-8})(x-5)} = \frac{1}{x-5}$$

Step 3: Set denominator of the simplified expression $=0$ to find the Vertical asymptote(s):

$$x-5=0 \rightarrow x=5. \quad (\text{V. A.})$$

Step 4: Set the canceled factor(s) $=0$ to find x location of hole(s):

$$x-8=0 \rightarrow x=8 \quad (\text{Hole})$$

19) degree of top $= 2$

Degree of bottom $= 2$.

Since degree of top $=$ degree of bottom, Horizontal asymptote is

$$y = \frac{\text{leading coeff. of top}}{\text{leading coeff. of bottom}} = \frac{10}{5} = 2$$

So, H.A. : $\boxed{y=2}$

20) Degree of top $= 3$, Degree of bottom $= 2$.

Since degree of top $>$ degree of bottom, there is NO horizontal asymptote.