

Student: _____
Date: _____

Course: Trigonometry

Test 3 covers Section 5.1, 5.2, 5.3, 5.4, 5.5

1. Choose the expression that completes an identity.

$$\tan^2 x + 1 = (1) \underline{\hspace{2cm}}$$

- (1) ☐ $\csc^2 x$ ☐ $\sec^2 x$
☐ $\cot x$ ☐ $\sin^2 x + \cos^2 x$
☐ $\frac{\sin x}{\cos x}$
☐ $\cos x$

2. For the expression in Column I, choose the expression from Column II that completes an identity.

Column I

Column II

$$1 = (1) \underline{\hspace{2cm}}$$

- A. $\sin^2 x + \cos^2 x$
 B. $\cos x$
 C. $\sec^2 x$
 D. $\frac{\sin x}{\cos x}$
 E. $\cot x$

- (1) ☐ $\sin^2 x + \cos^2 x$ ☐ $\cot x$
☐ $\cos x$
☐ $\sec^2 x$
☐ $\frac{\sin x}{\cos x}$

3. Write the expression in terms of sine and cosine, and then simplify so that no quotients appear in the final expression.

$$-\cos^2 \theta (\sec^2 \theta - 1)$$

Choose the correct answer below.

- ☐ A. $\frac{1}{\cos^2 \theta}$ ☐ B. $\sin^2 \theta$
☐ C. $-\frac{1}{\cos^2 \theta}$ ☐ D. $-\sin^2 \theta$
☐ E. $\cot^2 \theta$ ☐ F. $\tan^2 \theta$

4. Write the expression in terms of sine and cosine, and then simplify the expression so that no quotients appear and all functions are of t only.

$$\sin t(\csc t - \sin t)$$

$$\sin t(\csc t - \sin t) = \underline{\hspace{2cm}}$$

5. Fill in the blanks to complete the fundamental identities.

$$\tan \theta = \frac{1}{\sin \theta} = \frac{\sin \theta}{\cos \theta}$$

6. Perform the indicated operation and simplify the result so that there are no quotients.

$$(8 + \sin t)^2 + \cos^2 t$$

The simplified form, with no quotients, of $(8 + \sin t)^2 + \cos^2 t$ is _____.
(Do not factor.)

7. Write the following expression as a single trigonometric function or a power of a trigonometric function.

$$\frac{\sec \beta \tan \beta}{\csc \beta}$$

Choose the correct answer below.

- ☐ $\frac{1}{\tan^2 \beta}$
- ☐ 1
- ☐ $\tan^2 \beta$
- ☐ $\frac{\sin^2 \beta}{\cos^2 \beta}$

8. Verify that the trigonometric equation is an identity.

$$(1 - \sin^2 \alpha)(1 + \sin^2 \alpha) = 2 \cos^2 \alpha - \cos^4 \alpha$$

Which of the following statements establishes the identity?

- ☐ A. $(1 - \sin^2 \alpha)(1 + \sin^2 \alpha) = \cot^2 \alpha (\csc^2 \alpha + 1) = \cot^2 \alpha (\cot^2 \alpha + 1 + 1)$
 $= 2 \cos^2 \alpha - \cos^4 \alpha$
- ☐ B. $(1 - \sin^2 \alpha)(1 + \sin^2 \alpha) = \sec^2 \alpha (\tan^2 \alpha - 1) = \sec^2 \alpha (\sec^2 \alpha - 1 - 1)$
 $= 2 \cos^2 \alpha - \cos^4 \alpha$
- ☐ C. $(1 - \sin^2 \alpha)(1 + \sin^2 \alpha) = \csc^2 \alpha (\cot^2 \alpha - 1) = \csc^2 \alpha (\csc^2 \alpha - 1 - 1)$
 $= 2 \cos^2 \alpha - \cos^4 \alpha$
- ☐ D. $(1 - \sin^2 \alpha)(1 + \sin^2 \alpha) = \cos^2 \alpha (1 + \sin^2 \alpha) = \cos^2 \alpha (1 + 1 - \cos^2 \alpha)$
 $= 2 \cos^2 \alpha - \cos^4 \alpha$

9. Use identities to reduce the expression to a function of α .

$$\cos(\alpha + 0^\circ)$$

Choose the correct function for $\cos(\alpha + 0^\circ)$.

- ☐ A. $-\cos \alpha$
- ☐ B. $\sin \alpha$
- ☐ C. $-\sin \alpha$
- ☐ D. $\cos \alpha$

10. Use the cosine of a sum and cosine of a difference identities to find $\cos(s + t)$ and $\cos(s - t)$.

$$\sin s = -\frac{3}{5} \text{ and } \sin t = \frac{5}{13}, s \text{ in quadrant IV and } t \text{ in quadrant II}$$

$$\cos(s + t) = \underline{\hspace{2cm}}$$

(Simplify your answer, including any radicals. Use integers or fractions for any numbers in the expression.)

$$\cos(s - t) = \underline{\hspace{2cm}}$$

(Simplify your answer, including any radicals. Use integers or fractions for any numbers in the expression.)

11. Tell whether the statement is true or false.

$$\cos 39^\circ = \cos 30^\circ \cos 9^\circ - \sin 30^\circ \sin 9^\circ$$

Is the statement true or false?

- ☐ False
- ☐ True

12. Establish the identity.

$$\cos(2\pi + x) = \cos x$$

Choose the sequence of steps below that verifies the identity.

- ☐ A. $\cos(2\pi + x) = \cos 2\pi \cos x - \sin 2\pi \sin x = (1) \cos x - (0) \sin x = \cos x$
- ☐ B. $\cos(2\pi + x) = \cos \pi \cos x - \sin \pi \sin x = (1) \cos x - (0) \sin x = \cos x$
- ☐ C. $\cos(2\pi + x) = \cos \pi \cos x + \sin \pi \sin x = (-1) \cos x + (0) \sin x = \cos x$
- ☐ D. $\cos(2\pi + x) = \cos 2\pi \cos x + \sin 2\pi \sin x = (1) \cos x + (0) \sin x = \cos x$

13. Find the expression that is equivalent to $\sin 80^\circ \cos 30^\circ + \cos 80^\circ \sin 30^\circ$.

Choose the correct answer below.

- ☐ $\cos 50^\circ$
- ☐ $\cos 110^\circ$
- ☐ $\sin 110^\circ$
- ☐ $\sin 50^\circ$

14.

Find the expression that is equivalent to $\frac{\tan \frac{4\pi}{5} - \tan \frac{\pi}{6}}{1 + \tan \frac{4\pi}{5} \tan \frac{\pi}{6}}$.

Choose the correct answer below.

- ☐ A. $\tan \frac{19\pi}{30}$
- ☐ B. $\tan \frac{4\pi}{5} - \tan \frac{\pi}{6}$
- ☐ C. $\tan \frac{29\pi}{30}$
- ☐ D. $\tan \frac{2\pi}{15}$

15. Write the expression as a function of x , with no angle measure involved.

$$\sin \left(\frac{\pi}{4} + x \right)$$

$$\sin \left(\frac{\pi}{4} + x \right) = \underline{\hspace{2cm}}$$

(Simplify your answer, including any radicals. Use integers or fractions for any numbers in the expression.)

16. Establish the identity.

$$\frac{\sin(\alpha - \beta)}{\cos \alpha \cos \beta} = \tan \alpha - \tan \beta$$

Choose the sequence of steps below that verifies the identity.

- ☐ A. $\frac{\sin(\alpha - \beta)}{\cos \alpha \cos \beta} = \frac{\sin \alpha \cos \beta - \cos \alpha \sin \beta}{\sin \alpha \sin \beta} = \frac{\sin \alpha \cos \beta}{\sin \alpha \sin \beta} - \frac{\cos \alpha \sin \beta}{\sin \alpha \sin \beta} = \tan \alpha - \tan \beta$
- ☐ B. $\frac{\sin(\alpha - \beta)}{\cos \alpha \cos \beta} = \frac{\sin \alpha \sin \beta + \cos \alpha \cos \beta}{\cos \alpha \cos \beta} = \frac{\sin \alpha \sin \beta}{\cos \alpha \cos \beta} + \frac{\cos \alpha \cos \beta}{\cos \alpha \cos \beta} = \tan \alpha - \tan \beta$
- ☐ C. $\frac{\sin(\alpha - \beta)}{\cos \alpha \cos \beta} = \frac{\sin \alpha \cos \beta - \cos \alpha \sin \beta}{\cos \alpha \cos \beta} = \frac{\sin \alpha \cos \beta}{\cos \alpha \cos \beta} - \frac{\cos \alpha \sin \beta}{\cos \alpha \cos \beta} = \tan \alpha - \tan \beta$
- ☐ D. $\frac{\sin(\alpha - \beta)}{\cos \alpha \cos \beta} = \frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\cos \alpha \cos \beta} = \frac{\sin \alpha \cos \beta}{\cos \alpha \cos \beta} + \frac{\cos \alpha \sin \beta}{\cos \alpha \cos \beta} = \tan \alpha - \tan \beta$

17. Given that $\cos \theta = -\frac{60}{61}$ and $\sin \theta > 0$, determine the values of the sine and cosine functions for 2θ .

$$\sin 2\theta = \underline{\hspace{2cm}} \text{ (Type an integer or a simplified fraction.)}$$

$$\cos 2\theta = \underline{\hspace{2cm}} \text{ (Type an integer or a simplified fraction.)}$$

18. Use an identity to simplify the following expression.

$$\frac{\tan 10^\circ}{1 - \tan^2 10^\circ}$$

Choose the correct expression equal to $\frac{\tan 10^\circ}{1 - \tan^2 10^\circ}$ below.

☐ A. $\frac{1}{2} \tan 20^\circ$

☐ B. $\sin 20^\circ$

☐ C. $\frac{1}{2} \tan 10^\circ$

☐ D. $\frac{1}{2} \cos 10^\circ$

19. Use an identity to simplify the expression. Do not use a calculator.

$$\cos^2\left(\frac{\pi}{7}\right) - \sin^2\left(\frac{\pi}{7}\right)$$

$$\cos^2\left(\frac{\pi}{7}\right) - \sin^2\left(\frac{\pi}{7}\right) = \underline{\hspace{2cm}}$$

(Type an exact answer, using π as needed.)

20. Write the expression as a sum or difference of trigonometric functions.

$$6 \sin 4x \sin 6x$$

$$6 \sin 4x \sin 6x = \underline{\hspace{2cm}}$$

1. (1) $\sec^2 x$

2. (1) $\sin^2 x + \cos^2 x$

3. D. $-\sin^2 \theta$

4. $\cos^2 t$

5. $\cot \theta$

$\cos \theta$

6. $65 + 16 \sin t$

7. $\tan^2 \beta$

8. D. $(1 - \sin^2 \alpha)(1 + \sin^2 \alpha) = \cos^2 \alpha(1 + \sin^2 \alpha) = \cos^2 \alpha(1 + 1 - \cos^2 \alpha) = 2\cos^2 \alpha - \cos^4 \alpha$

9. D. $\cos \alpha$

10. $-\frac{33}{65}$

$-\frac{63}{65}$

11. True

12. A. $\cos(2\pi + x) = \cos 2\pi \cos x - \sin 2\pi \sin x = (1)\cos x - (0)\sin x = \cos x$

13. $\sin 110^\circ$

14. A. $\tan \frac{19\pi}{30}$

15. $\frac{\sqrt{2} \cos x + \sqrt{2} \sin x}{2}$

16. C. $\frac{\sin(\alpha - \beta)}{\cos \alpha \cos \beta} = \frac{\sin \alpha \cos \beta - \cos \alpha \sin \beta}{\cos \alpha \cos \beta} = \frac{\sin \alpha \cos \beta}{\cos \alpha \cos \beta} - \frac{\cos \alpha \sin \beta}{\cos \alpha \cos \beta} = \tan \alpha - \tan \beta$

17. $-\frac{1320}{3721}$

$$\frac{3479}{3721}$$

18. A. $\frac{1}{2} \tan 20^\circ$

19. $\cos \frac{2\pi}{7}$

20. $3 \cos 2x - 3 \cos 10x$
