① $\tan^2 x + 1 = \sec^2 x$ (This is a fundamental Pythagorean identity)

② $1 = \sin^2 x + \cos^2 x$ (This is a fundamental Pythagorean identity)

③

$$-\cos^2\theta\left(\sec^2\theta - 1\right) = -\cos^2\theta\left(\frac{1}{\cos^2\theta} - 1\right)$$

(Distribute)

Replace $1 = \sin^2\theta + \cos^2\theta$

$$= -\cancel{\cos^2\theta}\cdot\frac{1}{\cancel{\cos^2\theta}} + \cos^2\theta$$

$$= -1 + \cos^2\theta$$

$$= -\left(\sin^2\theta + \cos^2\theta\right) + \cos^2\theta$$

$$= -\sin^2\theta - \cancel{\cos^2\theta} + \cancel{\cos^2\theta}$$

$$= \boxed{-\sin^2\theta}$$

(Distribute)

④

$$\sin t\left(\csc t - \sin t\right) = \sin t\cdot\left(\frac{1}{\sin t} - \sin t\right)$$

$$= \sin t\cdot\frac{1}{\sin t} - \sin^2 t$$

$$= 1 - \sin^2 t = \boxed{\cos^2 t}$$

(b/c $\sin^2 t + \cos^2 t = 1$)

⑤ $\tan\theta = \dfrac{1}{\cot\theta} = \dfrac{\sin\theta}{\cos\theta}$ (Fundamental quotient identity)

⑥

$$(8 + \sin t)^2 + \cos^2 t = 64 + 16\sin t + \underbrace{\sin^2 t + \cos^2 t}_{1}$$

Distribute

$$= \boxed{65 + 16\sin t}$$

⑦

$$\frac{\sec\beta\cdot\tan\beta}{\csc\beta} = \frac{\frac{1}{\cos\beta}\cdot\frac{\sin\beta}{\cos\beta}}{\frac{1}{\sin\beta}} = \frac{\frac{\sin\beta}{\cos^2\beta}}{\frac{1}{\sin\beta}} = \frac{\sin\beta}{\cos^2\beta}\cdot\frac{\sin\beta}{1} = \frac{\sin^2\beta}{\cos^2\beta} = \boxed{\tan^2\beta}$$

(7) $\dfrac{[illegible]}{\csc\beta} = \dfrac{[illegible]}{\frac{1}{\sin\beta}} = \dfrac{[illegible]}{\frac{1}{\sin\beta}} = \dfrac{[illegible]}{\cos^2\beta}\cdot\dfrac{[illegible]}{1} = \dfrac{[illegible]}{\cos^2\beta} = \boxed{\tan\beta}$

quotient identities

Simplify

Simplify

Simplify

(8) $(1-\sin^2\alpha)(1+\sin^2\alpha) = \cos^2\alpha\cdot(1+1-\cos^2\alpha) = \cos^2\alpha\,(2-\cos^2\alpha)$

$= \boxed{2\cos^2\alpha - \cos^4\alpha}$

use $1-\sin^2\alpha = \cos^2\alpha$

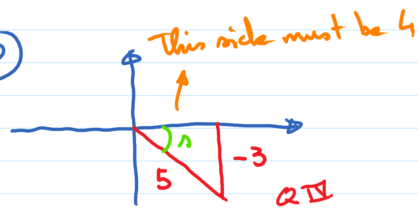
Distribute

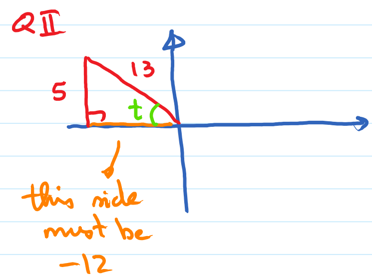
identity

(9) $\cos(\alpha+0^\circ) = \cos\alpha\underbrace{\cos 0^\circ}_{1} - \sin\alpha\underbrace{\sin 0^\circ}_{0}$

$= \boxed{\cos\alpha}$

(10)





So, $\sin s = -\frac{3}{5}$ and $\cos s = \frac{4}{5}$

So, $\sin t = \frac{5}{13}$ and $\cos t = -\frac{12}{13}$.

So, $\cos(s+t) = \cos s\cos t - \sin s\sin t = \frac{4}{5}\cdot\left(-\frac{12}{13}\right) - \left(-\frac{3}{5}\right)\left(\frac{5}{13}\right)$

$= \boxed{-\frac{33}{65}}$

$\cos(s-t) = \cos s\cos t + \sin s\sin t = \frac{4}{5}\left(-\frac{12}{13}\right) + \left(-\frac{3}{5}\right)\left(\frac{5}{13}\right)$

$= \boxed{-\frac{63}{65}}$

(11) True b/c of the identity $\cos(39^\circ) = \cos(30^\circ+9^\circ)$

$= \cos 30^\circ\cos 9^\circ - \sin 30^\circ\sin 9^\circ$

Identity

(12) $\cos(2\pi + x) = \cos 2\pi\cos x - \sin 2\pi\sin x$

12 $\cos(2\pi + x) \overset{\text{Identity}}{=} \underbrace{\cos 2\pi}_{1} \cos x - \underbrace{\sin 2\pi}_{0} \sin x$

$= \boxed{\cos x}$

13 $\sin 80^\circ \cos 30^\circ + \cos 80^\circ \sin 30^\circ \overset{\text{Identity}}{=} \sin(80^\circ + 30^\circ) = \boxed{\sin(110^\circ)}$

14 $\dfrac{\tan\frac{4\pi}{5} - \tan\frac{\pi}{6}}{1 + \tan\frac{4\pi}{5}\tan\frac{\pi}{6}} \overset{\text{Identity}}{=} \tan\left(\frac{4\pi}{5} - \frac{\pi}{6}\right) = \tan\left(\frac{24\pi}{30} - \frac{5\pi}{30}\right)$

$= \boxed{\tan\left(\frac{19\pi}{30}\right)}$

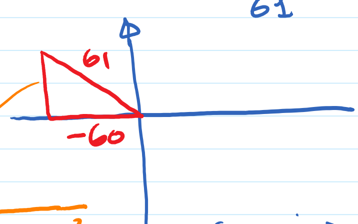
15 $\sin\left(\frac{\pi}{4} + x\right) \overset{\text{Identity}}{=} \sin\frac{\pi}{4}\cos x + \sin x \cos\frac{\pi}{4}$

$= \frac{\sqrt{2}}{2}\cos x + \frac{\sqrt{2}}{2}\sin x$

16 $\dfrac{\sin(\alpha-\beta)}{\cos\alpha\cos\beta} \overset{\text{Identity}}{=} \dfrac{\sin\alpha\cos\beta - \sin\beta\cos\alpha}{\cos\alpha\cos\beta} \overset{\text{Split}}{=} \dfrac{\sin\alpha\cancel{\cos\beta}}{\cos\alpha\cancel{\cos\beta}} - \dfrac{\sin\beta\cancel{\cos\alpha}}{\cancel{\cos\alpha}\cos\beta}$

$= \dfrac{\sin\alpha}{\cos\alpha} - \dfrac{\sin\beta}{\cos\beta} = \tan\alpha - \tan\beta.$

17 $\cos\theta = -\frac{60}{61}$; $\sin\theta > 0 \rightarrow \theta$ is in Q II.



$\sqrt{61^2 - (-60)^2}$
$= \sqrt{121} = 11$

So, $\sin\theta = \frac{11}{61}$

So, $\sin 2\theta \overset{\text{Identity}}{=} 2\sin\theta\cos\theta = 2\cdot\frac{11}{61}\cdot\left(-\frac{60}{61}\right)$

$= \boxed{-\frac{1320}{3721}}$

$\cos 2\theta \overset{\text{Identity}}{=} \cos^2\theta - \sin^2\theta = \left(-\frac{60}{61}\right)^2 - \left(\frac{11}{61}\right)^2$

$= \boxed{\frac{3479}{3721}}$

18 $2 \cdot \tan 10^\circ$ Identity ... So $\tan 10^\circ$ ...

18 $\dfrac{2 \cdot \tan 10^\circ}{1 - \tan^2 10^\circ} \overset{\text{Identity}}{=} \tan(20^\circ)$. So, $\dfrac{\tan 10^\circ}{1 - \tan^2 10^\circ} = \dfrac{1}{2}\tan(20^\circ)$

Double-angle identity

19 $\cos^2\left(\frac{\pi}{7}\right) - \sin^2\left(\frac{\pi}{7}\right) = \cos\left(\frac{2\pi}{7}\right)$

20 $6 \sin 4x \sin 6x = 6 \cdot \frac{1}{2}\left[\cos(-2x) - \cos(10x)\right] = 3\left(\cos(2x) - \cos(10x)\right)$

product to sum identity

Cosine of negative angle = cosine of angle