Practice Final Solution

(1) $\underbrace{y = \cos^{-1} x}_{\text{Inverse Cosine}}$ means that $x = \boxed{\cos y}$, for $0 \leq y \leq \pi$

(2) $\sin(\boxed{2\theta - 15^\circ}) = \cos(\boxed{3\theta - 20^\circ})$

Using complementary angles; we have:

$$(2\theta - 15^\circ) + (3\theta - 20^\circ) = 90^\circ$$

So, $5\theta - 35^\circ = 90^\circ$.

Hence, $5\theta = 125^\circ$

Thus, $\boxed{\theta = 25^\circ}$

(3) $\cos\theta = \text{x-coordinate} = -\dfrac{5}{13}$

$\sin\theta = \text{y-coordinate} = \dfrac{12}{13}$

$\sec\theta = \dfrac{1}{\cos\theta} = -\dfrac{13}{5}$

$\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{13}{12}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{12/13}{-5/13} = -\dfrac{12}{5}$

$\cot\theta = \dfrac{1}{\tan\theta} = -\dfrac{5}{12}$

④ $a^2 + b^2 = c^2 \rightarrow c^2 = 64 + 225$

$\rightarrow c^2 = 289 \rightarrow \boxed{c = 17.}$

$$\sin B = \frac{\text{opp. of } B}{\text{hyp.}} = \frac{b}{c} = \boxed{\frac{15}{17}}$$

$$\cos B = \frac{\text{adj. of } B}{\text{hyp.}} = \frac{a}{c} = \boxed{\frac{8}{17}}$$

$$\tan B = \frac{\text{opp. } B}{\text{adj. } B} = \frac{b}{a} = \boxed{\frac{15}{8}}$$

$$\sec B = \frac{1}{\cos B} = \boxed{\frac{17}{8}}$$

$$\csc B = \frac{1}{\sin B} = \boxed{\frac{17}{15}}$$

$$\cot B = \frac{1}{\tan B} = \boxed{\frac{8}{15}}$$

⑤ $\cos(x-y) = \cos x \cos y + \sin x \sin y$

cosine of a difference formula

⑥ $$\frac{-1 + \sec^2\alpha}{\tan\alpha} = \frac{\tan^2\alpha}{\tan\alpha} = \tan\alpha$$

Divide out the common factor (that is $\tan\alpha$)

A Pythagorean Identity

$\sec^2\alpha - 1 = \tan^2\alpha$

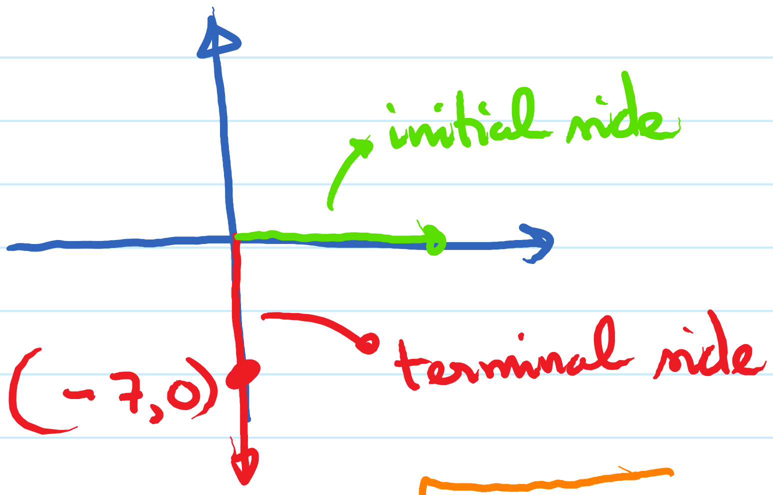
(Equivalently, $\tan^2\alpha + 1 = \sec^2\alpha$)

(7) Degree $\longrightarrow$ Radian:

Multiply by $\frac{\pi}{180}$

So, $630 \cdot \frac{\pi}{180} = \boxed{\frac{7\pi}{2}}$

(8)



$R = \sqrt{(-7)^2 + 0^2} = 7$

$\sin\theta = \frac{y}{R} = \frac{0}{7} = \boxed{0}$

$\cos\theta = \frac{x}{R} = \frac{-7}{7} = \boxed{-1}$

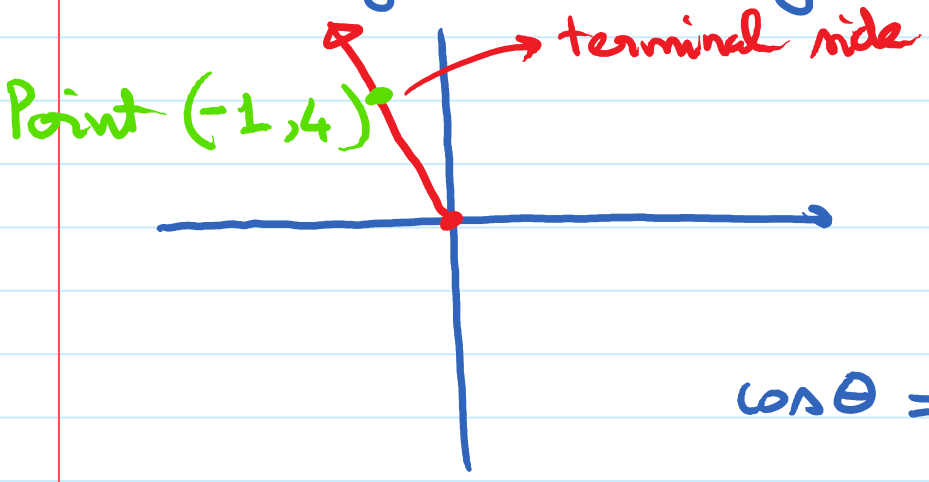
$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{0}{-1} = \boxed{0}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{0} = \boxed{\text{undefined}}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{-1} = \boxed{-1}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{0} = \boxed{\text{undefined}}$

(9) $4x + y = 0 \longrightarrow y = -4x$



$R = \sqrt{(-1)^2 + (4)^2} = \sqrt{17}$

$\sin\theta = \frac{y}{R} = \frac{4}{\sqrt{17}} = \boxed{\frac{4\sqrt{17}}{17}}$

$\cos\theta = \frac{x}{R} = \frac{-1}{\sqrt{17}} = \boxed{\frac{-\sqrt{17}}{17}}$

$\tan\theta = \frac{y}{x} = \frac{4}{-1} = \boxed{-4}$

$\csc\theta = \frac{1}{\sin\theta} = \boxed{\frac{\sqrt{17}}{4}}$; $\sec\theta = \frac{1}{\cos\theta} = -\sqrt{17}$

$\cot\theta = \frac{1}{\tan\theta} = \boxed{-\frac{1}{4}}$

(10) $\sin\theta = \text{y-coordinate} = \boxed{-\frac{4}{5}}$

$\cos\theta = \text{x-coordinate} = \boxed{-\frac{3}{5}}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-4/5}{-3/5} = \boxed{\frac{4}{3}}$

$\sec\theta = \frac{1}{\cos\theta} = \boxed{-\frac{5}{3}}$

$\csc\theta = \frac{1}{\sin\theta} = \boxed{-\frac{5}{4}}$

$\cot\theta = \frac{1}{\tan\theta} = \boxed{\frac{3}{4}}$

(11) By the Law of Cosines:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c^2 = 49 + 25 - 2\cdot5\cdot7\cdot\cos 98^\circ$$

$$c = \sqrt{74 - 70\cos 98^\circ} \simeq \boxed{9.2}$$

By the Law of Sines:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

So, $\sin A = \frac{a\cdot\sin C}{c} = \frac{7\cdot\sin 98^\circ}{9.2}$

$$\rightarrow A = \sin^{-1}\left(\frac{7\cdot\sin 98^\circ}{9.2}\right) \simeq \boxed{49^\circ}$$

$$\sin B = \frac{b\cdot\sin C}{c} = \frac{5\cdot\sin 98^\circ}{9.2}$$

$$\rightarrow B = \sin^{-1}\left(\frac{5\cdot\sin 98^\circ}{9.2}\right) \simeq \boxed{33^\circ}$$