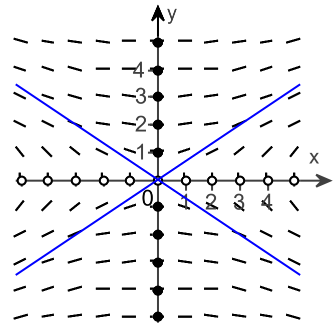
1. The direction field for $\frac{dy}{dx} = \frac{4x}{9y}$ is shown to the right.



(a) Verify that the straight lines $y = \pm\frac{2}{3}x$ are solution curves, provided $x \neq 0$.

(b) Sketch the solution curve with initial condition $y(0) = -1$.
(c) Sketch the solution curve with initial condition $y(2) = 1$.
(d) What can be said about the behavior of the above solutions as $x \rightarrow +\infty$? How about $x \rightarrow -\infty$?

(a) The restriction $y \neq 0$ is needed because $\frac{4x}{9y}$ is not defined when $y = 0$. Consider the straight lines one at a time. First let $y = \frac{2}{3}x$. In this case, the standard rules of differentiation yield $\frac{dy}{dx} =$ ____________.
(Simplify your answer.)

Substituting the expression for y into the differential equation yields $\frac{dy}{dx} = \frac{4x}{9(\_\_\_\_\_\_\_\_)}$.

The result from the previous step simplifies to ____________, verifying that the straight line $y = \frac{2}{3}x$ is a solution curve.
(Simplify your answer.)

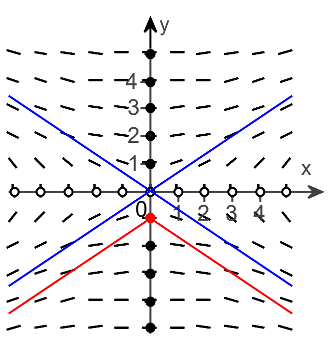
Now let $y = -\frac{2}{3}x$. In this case, the standard rules of differentiation yield $\frac{dy}{dx} =$ ____________.
(Simplify your answer.)

Substituting the expression for y into the differential equation yields $\frac{dy}{dx} = \frac{4x}{9(\_\_\_\_\_\_\_\_)}$.

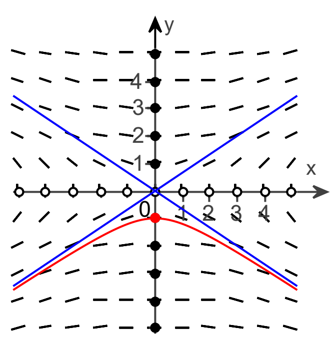
The result from the previous step simplifies to ____________, verifying that the straight line $y = -\frac{2}{3}x$ is a solution curve.
(Simplify your answer.)

(b) Which of the following shows the solution curve with initial condition $y(0) = -1$?

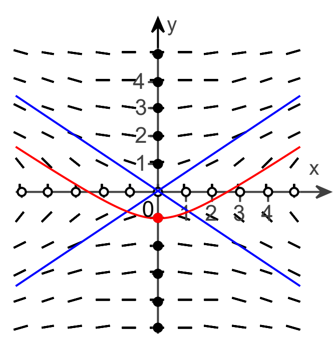
○ **A.**



○ **B.**

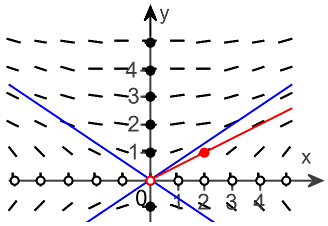


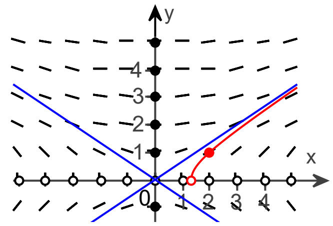
○ **C.**

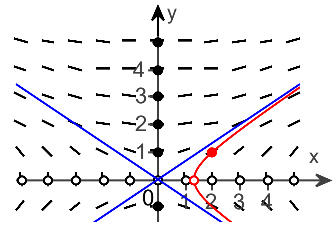


(c) Which of the following shows the solution curve with initial condition $y(2) = 1$?

○ **A.** ○ **B.** ○ **C.**







(d) What can be said about the behavior of the solution in part (b) as $x \to +\infty$? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** As $x \to +\infty$, the solution approaches _______ and has the line $y = \frac{2}{3}x$ as an asymptote.
- **B.** As $x \to +\infty$, the solution approaches _______ and has the line $y = -\frac{2}{3}x$ as an asymptote.
- **C.** The solution does not approach a single value or $\pm\infty$ as $x \to +\infty$.
- **D.** The solution does not exist for positive x.

What can be said about the behavior of the solution in part (b) as $x \to -\infty$? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** As $x \to -\infty$, the solution approaches _______ and has the line $y = \frac{2}{3}x$ as an asymptote.
- **B.** As $x \to -\infty$, the solution approaches _______ and has the line $y = -\frac{2}{3}x$ as an asymptote.
- **C.** The solution does not approach a single value or $\pm\infty$ as $x \to -\infty$.
- **D.** The solution does not exist for negative x.

What can be said about the behavior of the solution in part (c) as $x \to +\infty$? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** As $x \to +\infty$, the solution approaches _______ and has the line $y = -\frac{2}{3}x$ as an asymptote.
- **B.** As $x \to +\infty$, the solution approaches _______ and has the line $y = \frac{2}{3}x$ as an asymptote.
- **C.** The solution does not approach a single value or $\pm\infty$ as $x \to +\infty$.
- **D.** The solution does not exist for positive x.

What can be said about the behavior of the solution in part (c) as $x \to -\infty$? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** As $x \to -\infty$, the solution approaches _______ and has the line $y = \frac{2}{3}x$ as an asymptote.
- **B.** As $x \to -\infty$, the solution approaches _______ and has the line $y = -\frac{2}{3}x$ as an

asymptote.

- **C.** The solution does not approach a single value or $\pm\infty$ as $x \to -\infty$.
- **D.** The solution does not exist for negative x.

---

2. Determine whether the equation is exact. If it is, then solve it.

$$(2xy+8)dx + \left(x^2-8\right)dy = 0$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The equation is exact and an implicit solution in the form $F(x,y) = C$ is __________ $= C$, where C is an arbitrary constant.
  (Type an expression using x and y as the variables.)
- **B.** The equation is not exact.

---

3. A differential equation is given. Classify it as an ordinary differential equation (ODE) or a partial differential equation (PDE), give the order, and indicate the independent and dependent variables. If the equation is an ordinary differential equation, indicate whether the equation is linear or nonlinear.

$$9\frac{d^4t}{dq^4} + 4\frac{dt}{dq} + 7t = 4\cos 5q$$

Classify the given differential equation. Choose the correct answer below.

- linear ordinary differential equation
- nonlinear ordinary differential equation
- partial differential equation

The order of the differential equation is __________. (Type a whole number.)

The independent (1) __________ and the dependent (2) __________

(1) variable is q / variable is t
(2) variable is q. / variable is t.

---

4. Solve the initial value problem.

$$\frac{dy}{dx} + 4y - 7e^{-x} = 0, \quad y(0) = -\frac{2}{3}$$

The solution is $y(x) =$ __________.

5. A differential equation is given along with the field or problem area in which it arises. Classify it as an ordinary differential equation (ODE) or a partial differential equation (PDE), give the order, and indicate the independent and dependent variables. If the equation is an ordinary differential equation, indicate whether the equation is linear or nonlinear.

$$\sqrt{1-y}\,\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = 0$$

(Kidder's equation, flow of gases through a porous medium)

Classify the given differential equation. Choose the correct answer below.

- ○ linear ordinary differential equation
- ○ partial differential equation
- ○ nonlinear ordinary differential equation

The order of the differential equation is ____________.
(Type a whole number.)

The independent (1) ____________ and the dependent (2) ____________

(1) ○ variable is x
○ variable is y

(2) ○ variable is x.
○ variable is y.

6. Classify the equation as separable, linear, exact, or none of these. Note that it is possible for the equation to have more than one classification.

$$x^4ydx + 9dy = 0$$

Select all that apply.

- ☐ **A.** Separable
- ☐ **B.** Exact
- ☐ **C.** Linear
- ☐ **D.** None of these

7. Obtain the general solution to the equation.

$$y\frac{dx}{dy} + 2x = 8y^5$$

The general solution is $x(y) =$ ____________, ignoring lost solutions, if any.

8. Solve the equation.

$$\frac{dy}{dx} = 3x^2\left(8 + y^2\right)^{\frac{3}{2}}$$

An implicit solution in the form $F(x,y) = C$ is ____________ $= C$, where C is an arbitrary constant.
(Type an expression using x and y as the variables.)

9. Determine whether the given function is a solution to the given differential equation.

$\theta = 4e^{3t} - 2e^{4t}$, $\frac{d^2\theta}{dt^2} - \theta\frac{d\theta}{dt} + 5\theta = -13e^{4t}$

The function $\theta = 4e^{3t} - 2e^{4t}$ (1) __________ a solution to the differential equation $\frac{d^2\theta}{dt^2} - \theta\frac{d\theta}{dt} + 5\theta = -13e^{4t}$, because when $4e^{3t} - 2e^{4t}$ is substituted for θ, __________ is substituted for $\frac{d\theta}{dt}$ and __________ is substituted for $\frac{d^2\theta}{dt^2}$, the two sides of the differential equation (2) __________ equivalent on any intervals of t.

(1) ○ is
○ is not

(2) ○ are not
○ are

1. $\frac{2}{3}$

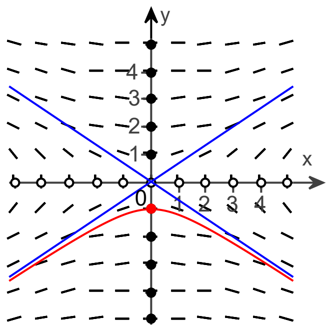
$\frac{2}{3}x$

$\frac{2}{3}$

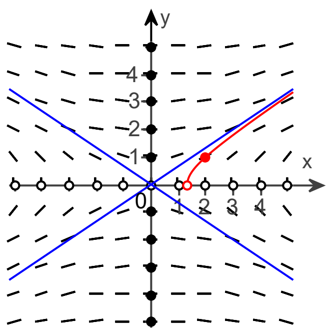
$-\frac{2}{3}$

$-\frac{2}{3}x$

$-\frac{2}{3}$



B.



B.

B. As $x \to +\infty$, the solution approaches **$-\infty$** and has the line $y = -\frac{2}{3}x$ as an asymptote.

A. As $x \to -\infty$, the solution approaches **$-\infty$** and has the line $y = \frac{2}{3}x$ as an asymptote.

B. As $x \to +\infty$, the solution approaches **$+\infty$** and has the line $y = \frac{2}{3}x$ as an asymptote.

D. The solution does not exist for negative x.

---

2. A.

The equation is exact and an implicit solution in the form $F(x,y) = C$ is **$x^2y + 8x - 8y$** $= C$, where C is an arbitrary constant.
(Type an expression using x and y as the variables.)

---

3. linear ordinary differential equation

   4

   (1) variable is q

   (2) variable is t.

---

4. $\frac{7}{3} e^{-x} - 3 e^{-4x}$

---

5. nonlinear ordinary differential equation

   2

   (1) variable is x

   (2) variable is y.

---

6. A. Separable, C. Linear

---

7. $\frac{8}{7} y^5 + Cy^{-2}$

---

8. $\frac{y}{8\sqrt{8 + y^2}} - x^3$

---

9. (1) is not

   $12 e^{3t} - 8 e^{4t}$

   $36 e^{3t} - 32 e^{4t}$

   (2) are not
