1. Express the given power series as a series with generic term x^k.

$$\sum_{n=6}^{\infty} n(n-2)a_n x^{n+6}$$

$$\sum_{k=\_\_\_\_\_}^{\infty} \_\_\_\_\_$$

2. Use the annihilator method to determine the form of a particular solution for the given equation.

$y'' + 17y' + 72y = e^{6x} - \sin x$

Find a differential operator that will annihilate the nonhomogeneity $e^{6x} - \sin x$.

_____

(Type the lowest-order annihilator that contains the minimum number of terms. Type your answer in factored or expanded form.)

What is the form of the particular solution?

$y_p(x) =$ _____

3. Determine the first three nonzero terms in the Taylor polynomial approximation for the given initial value problem.

$y' = \sin y + 2e^{5x}; y(0) = 0$

The Taylor approximation to three nonzero terms is $y(x) =$ _____ + ⋯.

4. Find a power series expansion about $x = 0$ for a general solution to the given differential equation. Your answer should include a general formula for the coefficients.

$y' - 2xy = 0$

$$y = a_0 \sum_{n=0}^{\infty} (\_\_\_\_\_)$$

5. Find a differential operator that annihilates the given function.

$e^{8x} - 16e^{x}$

A differential operator that annihilates $e^{8x} - 16e^{x}$ is _____.

(Type the lowest-order annihilator that contains the minimum number of terms. Type your answer in factored or expanded form.)

6. Find a general solution for the given differential equation with x as the independent variable.

$y^{(4)} + 6y'' + 9y = 0$

A general solution with x as the independent variable is $y(x) =$ _____.

7. Find a general solution for the given differential equation with x as the independent variable.

$$y''' - y'' - 24y' - 36y = 0$$

A general solution with x as the independent variable is $y(x) =$ _____ .

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8. Find the first four nonzero terms in a power series expansion about $x = 0$ for a general solution to the given differential equation.

$$\left(x^2 + 10\right) y'' + y = 0$$

$y(x) =$ _____ $+ \cdots$

(Type an expression in terms of a_0 and a_1 that includes all terms up to order 3.)

1. 12

$(k-6)(k-8)a_{k-6}x^k$

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2. $(D-6)\left(D^2+1\right)$

$c_3\, e^{6x} + c_4 \cos x + c_5 \sin x$

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3. $2x + 6x^2 + \frac{31}{3}x^3$

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4. $\frac{x^{2n}}{n!}$

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5. $(D-8)(D-1)$

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6. $C_1 \cos\sqrt{3}x + C_2 x \cos\sqrt{3}x + C_3 \sin\sqrt{3}x + C_4 x \sin\sqrt{3}x$

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7. $C_1\, e^{-2x} + C_2\, e^{6x} + C_3\, e^{-3x}$

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8. $a_0 + a_1 x - \frac{1}{20}a_0 x^2 - \frac{1}{60}a_1 x^3$
