1. Determine $\mathcal{L}^{-1}\{F\}$.

$$F(s) = \frac{5s^2 - 11s + 2}{s(s-5)(s-1)}$$

Click here to view the table of Laplace transforms.[1]
Click here to view the table of properties of Laplace transforms.[2]

$\mathcal{L}^{-1}\{F\} =$ ____________

1: Table of Laplace Transforms

f(t)	F(s) = $\mathcal{L}\{f\}(s)$
1	$\frac{1}{s}$, s > 0
e^{at}	$\frac{1}{s-a}$, s > 0
t^n, n = 1,2,...	$\frac{n!}{s^{n+1}}$, s > 0
sin bt	$\frac{b}{s^2+b^2}$, s > 0
cos bt	$\frac{s}{s^2+b^2}$, s > 0
$e^{at}t^n$, n = 1,2,...	$\frac{n!}{(s-a)^{n+1}}$, s > a
e^{at} **sin** bt	$\frac{b}{(s-a)^2+b^2}$, s > a
e^{at} **cos** bt	$\frac{s-a}{(s-a)^2+b^2}$, s > a

2: Properties of Laplace Transforms

$\mathcal{L}\{f+g\} = \mathcal{L}\{f\} + \mathcal{L}\{g\}$
$\mathcal{L}\{cf\} = c\mathcal{L}\{f\}$ for any constant c
$\mathcal{L}\{e^{at}f(t)\}(s) = \mathcal{L}\{f\}(s-a)$
$\mathcal{L}\{f'\}(s) = s\mathcal{L}\{f\}(s) - f(0)$
$\mathcal{L}\{f''\}(s) = s^2\mathcal{L}\{f\}(s) - sf(0) - f'(0)$
$\mathcal{L}\{f^{(n)}\}(s) = s^n\mathcal{L}\{f\}(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \cdots - f^{(n-1)}(0)$
$\mathcal{L}\{t^n f(t)\}(s) = (-1)^n \frac{d^n}{ds^n}(\mathcal{L}\{f\}(s))$
$\mathcal{L}^{-1}\{F_1 + F_2\} = \mathcal{L}^{-1}\{F_1\} + \mathcal{L}^{-1}\{F_2\}$
$\mathcal{L}^{-1}\{cF\} = c\mathcal{L}^{-1}\{F\}$

2. Use the Laplace transform table and the linearity of the Laplace transform to determine the following transform. Complete parts a and b below.

$$\mathcal{L}\left\{e^{4t}\sin 9t - t^2 + e^{6t}\right\}$$

[3] Click the icon to view the Laplace transform table.

a. Determine the formula for the Laplace transform.

$\mathcal{L}\left\{e^{4t}\sin 9t - t^2 + e^{6t}\right\}$ = ____________ (Type an expression using s as the variable.)

b. What is the restriction on s?

s > ____________ (Type an integer or a fraction.)

3: Data Table

Brief table of Laplace transforms	
f(t)	**F(s) = $\mathcal{L}$\{f\}(s)**
1	$\frac{1}{s}$, s > 0
e^{at}	$\frac{1}{s-a}$, s > a
t^n	$\frac{n!}{s^{n+1}}$, s > 0
sin bt	$\frac{b}{s^2+b^2}$, s > 0
cos bt	$\frac{s}{s^2+b^2}$, s > 0
$e^{at}t^n$	$\frac{n!}{(s-a)^{n+1}}$, s > a
e^{at} **sin** bt	$\frac{b}{(s-a)^2+b^2}$, s > a
e^{at} **cos** bt	$\frac{s-a}{(s-a)^2+b^2}$, s > a

3. Determine the inverse Laplace transform of the function below.

$$\frac{7s+42}{s^2+4s+20}$$

Click here to view the table of Laplace transforms.[4]
Click here to view the table of properties of Laplace transforms.[5]

$$\mathcal{L}^{-1}\left\{\frac{7s+42}{s^2+4s+20}\right\} = \_\_\_\_\_\_\_\_$$

4: Table of Laplace Transforms

f(t)	**F(s) = $\mathcal{L}$\{f\}(s)**
1	$\frac{1}{s}$, s > 0
e^{at}	$\frac{1}{s-a}$, s > 0
t^n, n = 1,2,...	$\frac{n!}{s^{n+1}}$, s > 0
sin bt	$\frac{b}{s^2+b^2}$, s > 0
cos bt	$\frac{s}{s^2+b^2}$, s > 0
$e^{at}t^n$, n = 1,2,...	$\frac{n!}{(s-a)^{n+1}}$, s > a
e^{at} **sin** bt	$\frac{b}{(s-a)^2+b^2}$, s > a
e^{at} **cos** bt	$\frac{s-a}{(s-a)^2+b^2}$, s > a

5: Properties of Laplace Transforms

$\mathcal{L}\{f+g\} = \mathcal{L}\{f\} + \mathcal{L}\{g\}$
$\mathcal{L}\{cf\} = c\mathcal{L}\{f\}$ for any constant c
$\mathcal{L}\{e^{at}f(t)\}(s) = \mathcal{L}\{f\}(s-a)$
$\mathcal{L}\{f'\}(s) = s\mathcal{L}\{f\}(s) - f(0)$
$\mathcal{L}\{f''\}(s) = s^2\mathcal{L}\{f\}(s) - sf(0) - f'(0)$
$\mathcal{L}\{f^{(n)}\}(s) = s^n\mathcal{L}\{f\}(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \cdots - f^{(n-1)}(0)$
$\mathcal{L}\{t^n f(t)\}(s) = (-1)^n\frac{d^n}{ds^n}(\mathcal{L}\{f\}(s))$
$\mathcal{L}^{-1}\{F_1+F_2\} = \mathcal{L}^{-1}\{F_1\} + \mathcal{L}^{-1}\{F_2\}$
$\mathcal{L}^{-1}\{cF\} = c\mathcal{L}^{-1}\{F\}$

4. Given that $\mathcal{L}\{\cos bt\}(s) = \frac{s}{s^2 + b^2}$, use the translation property to compute $\mathcal{L}\{e^{at}\cos bt\}$.

Click here to view the table of properties of Laplace transforms.[6]

$\mathcal{L}\{e^{at}\cos bt\}(s) =$ ____________

6: Properties of Laplace Transforms

$\mathcal{L}\{f + g\} = \mathcal{L}\{f\} + \mathcal{L}\{g\}$
$\mathcal{L}\{cf\} = c\mathcal{L}\{f\}$ for any constant c
$\mathcal{L}\{e^{at}f(t)\}(s) = \mathcal{L}\{f\}(s - a)$
$\mathcal{L}\{f'\}(s) = s\mathcal{L}\{f\}(s) - f(0)$
$\mathcal{L}\{f''\}(s) = s^2\mathcal{L}\{f\}(s) - sf(0) - f'(0)$
$\mathcal{L}\{f^{(n)}\}(s) = s^n\mathcal{L}\{f\}(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \cdots - f^{(n-1)}(0)$
$\mathcal{L}\{t^n f(t)\}(s) = (-1)^n \frac{d^n}{ds^n}(\mathcal{L}\{f\}(s))$

5. Solve for Y(s), the Laplace transform of the solution y(t) to the initial value problem below.

$y'' + 7y = 4t^3$, $y(0) = 0$, $y'(0) = 0$

Click here to view the table of Laplace transforms.[7]
Click here to view the table of properties of Laplace transforms.[8]

Y(s) = ____________

7: Table of Laplace Transforms

f(t)	**F(s) = $\mathcal{L}\{f\}(s)$**
1	$\frac{1}{s}$, s > 0
e^{at}	$\frac{1}{s-a}$, s > 0
t^n, n = 1,2,...	$\frac{n!}{s^{n+1}}$, s > 0
sin bt	$\frac{b}{s^2+b^2}$, s > 0
cos bt	$\frac{s}{s^2+b^2}$, s > 0
$e^{at}t^n$, n = 1,2,...	$\frac{n!}{(s-a)^{n+1}}$, s > a
e^{at} **sin** bt	$\frac{b}{(s-a)^2+b^2}$, s > a
e^{at} **cos** bt	$\frac{s-a}{(s-a)^2+b^2}$, s > a

8: Properties of Laplace Transforms

$\mathcal{L}\{f+g\}$	$= \mathcal{L}\{f\} + \mathcal{L}\{g\}$
$\mathcal{L}\{cf\}$	$= c\mathcal{L}\{f\}$ for any constant c
$\mathcal{L}\{e^{at}f(t)\}(s)$	$= \mathcal{L}\{f\}(s-a)$
$\mathcal{L}\{f'\}(s)$	$= s\mathcal{L}\{f\}(s) - f(0)$
$\mathcal{L}\{f''\}(s)$	$= s^2\mathcal{L}\{f\}(s) - sf(0) - f'(0)$
$\mathcal{L}\{f^{(n)}\}(s)$	$= s^n\mathcal{L}\{f\}(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \cdots - f^{(n-1)}(0)$
$\mathcal{L}\{t^n f(t)\}(s)$	$= (-1)^n \frac{d^n}{ds^n}(\mathcal{L}\{f\}(s))$
$\mathcal{L}^{-1}\{F_1 + F_2\}$	$= \mathcal{L}^{-1}\{F_1\} + \mathcal{L}^{-1}\{F_2\}$
$\mathcal{L}^{-1}\{cF\}$	$= c\mathcal{L}^{-1}\{F\}$

6. Solve the initial value problem below using the method of Laplace transforms.

$$y'' - 12y' + 52y = 164e^{t}, \; y(0) = 4, \; y'(0) = 8$$

Click here to view the table of Laplace transforms.[9]
Click here to view the table of properties of Laplace transforms.[10]

y(t) = ____________
(Type an exact answer in terms of e.)

9: Table of Laplace Transforms

f(t)	**F(s) = $\mathcal{L}$\{f\}(s)**
1	$\frac{1}{s}$, $s > 0$
e^{at}	$\frac{1}{s-a}$, $s > 0$
t^{n}, $n = 1,2,\ldots$	$\frac{n!}{s^{n+1}}$, $s > 0$
$\sin bt$	$\frac{b}{s^2+b^2}$, $s > 0$
$\cos bt$	$\frac{s}{s^2+b^2}$, $s > 0$
$e^{at}t^{n}$, $n = 1,2,\ldots$	$\frac{n!}{(s-a)^{n+1}}$, $s > a$
$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}$, $s > a$
$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}$, $s > a$

10: Properties of Laplace Transforms

$\mathcal{L}\{f+g\} = \mathcal{L}\{f\} + \mathcal{L}\{g\}$
$\mathcal{L}\{cf\} = c\mathcal{L}\{f\}$ for any constant c
$\mathcal{L}\{e^{at}f(t)\}(s) = \mathcal{L}\{f\}(s-a)$
$\mathcal{L}\{f'\}(s) = s\mathcal{L}\{f\}(s) - f(0)$
$\mathcal{L}\{f''\}(s) = s^2\mathcal{L}\{f\}(s) - sf(0) - f'(0)$
$\mathcal{L}\{f^{(n)}\}(s) = s^n\mathcal{L}\{f\}(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \cdots - f^{(n-1)}(0)$
$\mathcal{L}\{t^n f(t)\}(s) = (-1)^n \frac{d^n}{ds^n}(\mathcal{L}\{f\}(s))$
$\mathcal{L}^{-1}\{F_1 + F_2\} = \mathcal{L}^{-1}\{F_1\} + \mathcal{L}^{-1}\{F_2\}$
$\mathcal{L}^{-1}\{cF\} = c\mathcal{L}^{-1}\{F\}$

7.

Let f(t) be a function on [0, ∞). The Laplace transform of f is the function F defined by the integral $F(s) = \int_0^{\infty} e^{-st} f(t)\,dt$.

Use this definition to determine the Laplace transform of the following function.

$$f(t) = \begin{cases} 19 - t, & 0 < t < 19 \\ 0, & 19 < t \end{cases}$$

The Laplace transform of f(t) is F(s) = ____________ for s ≠ ____________ and $F(s) = \frac{361}{2}$ otherwise.

(Type exact answers.)

8. Use the accompanying tables of Laplace transforms and properties of Laplace transforms to find the Laplace transform of the function below.

$(t-2)^4$

Click here to view the table of Laplace transforms.[11]
Click here to view the table of properties of Laplace transforms.[12]

$\mathcal{L}\{(t-2)^4\} =$ ____________

11: Table of Laplace Transforms

f(t)	**F(s) = $\mathcal{L}$\{f\}(s)**
1	$\frac{1}{s}$, s > 0
e^{at}	$\frac{1}{s-a}$, s > 0
t^n, n = 1,2,...	$\frac{n!}{s^{n+1}}$, s > 0
sin bt	$\frac{b}{s^2+b^2}$, s > 0
cos bt	$\frac{s}{s^2+b^2}$, s > 0
$e^{at}t^n$, n = 1,2,...	$\frac{n!}{(s-a)^{n+1}}$, s > a
e^{at} **sin** bt	$\frac{b}{(s-a)^2+b^2}$, s > a
e^{at} **cos** bt	$\frac{s-a}{(s-a)^2+b^2}$, s > a

12: Properties of Laplace Transforms

$\mathcal{L}\{f+g\} = \mathcal{L}\{f\} + \mathcal{L}\{g\}$
$\mathcal{L}\{cf\} = c\mathcal{L}\{f\}$ for any constant c
$\mathcal{L}\{e^{at}f(t)\}(s) = \mathcal{L}\{f\}(s-a)$
$\mathcal{L}\{f'\}(s) = s\mathcal{L}\{f\}(s) - f(0)$
$\mathcal{L}\{f''\}(s) = s^2\mathcal{L}\{f\}(s) - sf(0) - f'(0)$
$\mathcal{L}\{f^{(n)}\}(s) = s^n\mathcal{L}\{f\}(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \cdots - f^{(n-1)}(0)$
$\mathcal{L}\{t^n f(t)\}(s) = (-1)^n \frac{d^n}{ds^n}(\mathcal{L}\{f\}(s))$

1. $\frac{2}{5} + \frac{18}{5} e^{5t} + e^{t}$

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2. $\frac{9}{(s-4)^2 + 81} - \frac{2}{s^3} + \frac{1}{s-6}$

6

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3. $7 e^{-2t} \cos 4t + 7 e^{-2t} \sin 4t$

---

4. $\frac{s-a}{(s-a)^2 + b^2}$

---

5. $\frac{24}{s^4 \left(s^2 + 7\right)}$

---

6. $e^{6t} \sin 4t + 4 e^{t}$

---

7. $\frac{e^{-19s} + 19s - 1}{s^2}$

0

---

8. $\frac{24}{s^5} - \frac{48}{s^4} + \frac{48}{s^3} - \frac{32}{s^2} + \frac{16}{s}$
